Reprinted from Journal of Applied Meteorology, Vol. 8, No. 5, October, 1969, pp. 815–819
American Meteorological Society
Printed in U. S. A.

A Method for Estimating Rainfall Rate-Radar Reflectivity Relationships

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(Manuscript received 24 December 1968, in revised form 23 June 1969)

Raindrop-size distributions obtained with the drop camera have been used to determine rainfall rate-radar reflectivity relationships for nine different locations throughout the world. Since the climates sampled were quite varied, an extrapolation of these *Z-R* relationships to other areas of the world with similar "drop-spectra climates" can be performed. Two climatic parameters, the mean annual per cent of rain days that are thunderstorm days, and the mean annual relative humidity at 0.5 km above ground, were found to be highly correlated with the coefficient *A* and exponent *b* in the *Z-R* equation, $Z=AR^b$. Regression equations based on the two climatic parameters were determined, permitting an estimation of the *Z-R* relationship for any area once the parameters are obtained.

1. Introduction

Raindrop spectra obtained with the drop camera, as described by Jones and Dean (1953) and Mueller (1960) for nine locations around the world, have been used to determine *Z-R* relationships for these areas (Stout and Mueller, 1968). The locations for the camera were chosen so that various climates would be sampled.

At two of these locations, synoptic stratification of the data was found to be effective in reducing the standard error of estimate (S.E.) around the *Z-R* regression line (Cataneo and Stout, 1968). Therefore, in these areas it is recommended that the appropriate *Z-R* equations be used for the various synoptic situations when estimating rainfall amounts with radar. In places where *Z-R* studies have not been performed, the question arises as to what equation is to be used for radar-rainfall determinations. It has often been the practice to use the Marshall-Palmer relationship ($Z=220R^{1.6}$) when no other is known. However, if the relationships for the nine drop camera locations were extrapolated to other areas of the world with similar "drop-spectra climates," then a better estimate of the *Z-R* equation may be obtained. The basic problem then would be to identify the variables determining the raindrop climates.

It was first believed that perhaps one of the standard schemes of climate classification, Köppen's (Haurwitz and Austin, 1944) for example, would divide the world into regions of similar drop-size spectra, hence similar *Z-R* relationships. However, this approach was not satisfactory, since most of the methods used for classifying climates are based on temperature and precipitation amounts in various combinations, and drop-size distribution variations are not dependent, to any appreciable degree, upon these parameters. For example, according to Köppen, both Island Beach, N. J., and Champaign, Ill., two of the drop camera locations, are classified as a Cfa climate, which is a warm, temperate, rainy climate without a dry season and with a hot summer. The *Z-R* equations for these areas are $Z=256^{1.41}$ for New Jersey and $Z=372R^{1.47}$ for Illinois. Since these suggest a substantial difference in the drop-size spectra for rains of similar rainfall rates, apparently the factors affecting the drop-size distributions are not defined satisfactorily by Köppen's classification method. A search for appropriate factors was therefore initiated.

2. Procedures

The first step in this investigation was to determine the parameters to be used for distinguishing the drop-size distribution areas. Concurrently, it was realized that it would be of little value to find a suitable parameter and then discover that the data associated with it were not available on a worldwide scale; it was essential that both conditions be met for the procedure to be practical.

For example, the concentration of condensation nuclei was considered as a parameter, but sufficient data were not available. Other parameters that were considered are thermodynamic instability, updraft and downdraft velocities, wind shear at cloud level, and some measure of coalescence and accretion. The field was narrowed down to three possible variables: 1) the mean per cent of the annual number of rain days that are thunderstorm days; 2) the mean annual relative humidity at 0.5 km (about 1600 ft) above ground level; and 3) the height of the mean annual freezing level above ground. All of these data are readily available worldwide.

The first variable was considered because previous analyses of the drop data collected from the drop camera locations indicated the presence of larger drops in thunderstorm rains, hence larger coefficients in the *Z-R* equations. Therefore, it followed that the greater the per cent of rain days that were thunderstorm days, the larger would be the coefficient of the average *Z-R*

TABLE 1. Correlation coefficients and standard errors of estimate of the *Z-R* regression parameters with climatic variables.

	Correlation coefficients		
$Z=AR^b$	Mean annual per cent of rain days that are thunderstorm days (1)	Mean annual relative humidity at 0.5 km above ground (2)	Mean annual freezing level in feet above ground (3)
A	0.59	−0.80	−0.30
b	0.06	−0.70	−0.52

	Multiple correlation with variables			Standard error of estimate with variables			
	(1) (2) & (3)	(1) & (2)	(2) & (3)	(1) (2) & (3)	(1) & (2)	(2) & (3)	(2)
A	0.91	0.91	0.84	59.4	55.3	70.5	73.3
b	0.70	0.70	0.70	0.094	0.087	0.086	0.081

relationship for a particular area (when the other variables remained constant).

The second was investigated because the relative humidity between cloud base and the ground affects the amount of evaporation of the raindrops as they fall from cloud base to the ground. The effects of evaporation on smaller drops are more pronounced than on larger ones, resulting in larger coefficients in the *Z-R* equation when compared with a situation where no evaporation is occurring on the raindrops (Mueller *et al.*, 1967).

The third parameter was considered because it appeared reasonable to assume that the height of the freezing level would be some indication of the amount of cloud growth that occurred above and below the freezing level, assuming that the average cloud base height for different areas remains relatively constant. If the mechanisms responsible for changes in drop-size distribution were dependent on how much of the precipitation formed by the ice or water process, then the third independent variable would have a direct bearing on the *Z-R* equation.

3. Analysis of data

In order to evaluate the importance of these three factors, the corresponding data for the nine sampled locations were obtained. The nine locations are: Miami, Fla.; Island Beach, N. J.; Franklin, N. C.; Champaign, Ill.; Corvallis, Ore.; Woody Island, Alaska; Majuro, Marshall Islands; Bogor, Indonesia; and Flagstaff, Ariz. Since *Z-R* relationships have been established for these areas, some measure of the effectiveness of the three variables can be determined.

The simple and multiple correlation coefficients between the three independent variables and the A and b values are shown in Table 1, along with the S.E. of A and b. The correlation between the first two independent variables and A was good when compared to freezing level, which was poorly correlated. This is further demonstrated by the decrease in the S.E. when freezing level is not included; the removal of an independent variable usually results in an increase in the S.E. The multiple correlation remains essentially unchanged, which also is an indication of the lack of effectiveness of freezing level as one of the independent variables. Because of this, it was decided not to use freezing level in the analysis. Also, since the correlation of b with thunderstorm days is rather poor, relative humidity alone was used as the predictor for b.

The S.E. of A (55.3) means that an error of less than ± 55.3 units on A can be expected 68% of the time; the S.E. of b is 0.081. The ranges of A and b in the data used are 372 and 0.32, respectively. With the nine points (locations) used, and with six degrees of freedom, a correlation of 0.906 (observed sample multiple correlation on A) is significant at the 99% level. A correlation of 0.701 (observed simple correlation of relative humidity with b) is significant at the 95% level.

The regression coefficients calculated from the nine-station analysis can be used to generate a family of lines for different values of relative humidity, using A as ordinate and thunderstorm days as abscissa; this allows a graphical determination of A for any combination of thunderstorm days and relative humidity. The same approach is used to determine b values using relative humidity (Fig. 1). The regression equations for A and b are

$$A=1.372(\mathrm{TD})-4.702(\mathrm{RH})+571, \qquad (1)$$

$$b=-0.00444(\mathrm{RH})+1.776, \qquad (2)$$

where TD is the mean annual per cent of rain days which are thunderstorm days, and RH is the mean annual relative humidity at 0.5 km above ground level.

The data associated with the two independent variables for the original nine locations were then used to determine the A and b values using Fig. 1. These values were then compared in Table 2 with the actual *Z-R* relationships established for these areas. The per cent thunderstorm days and relative humidity data

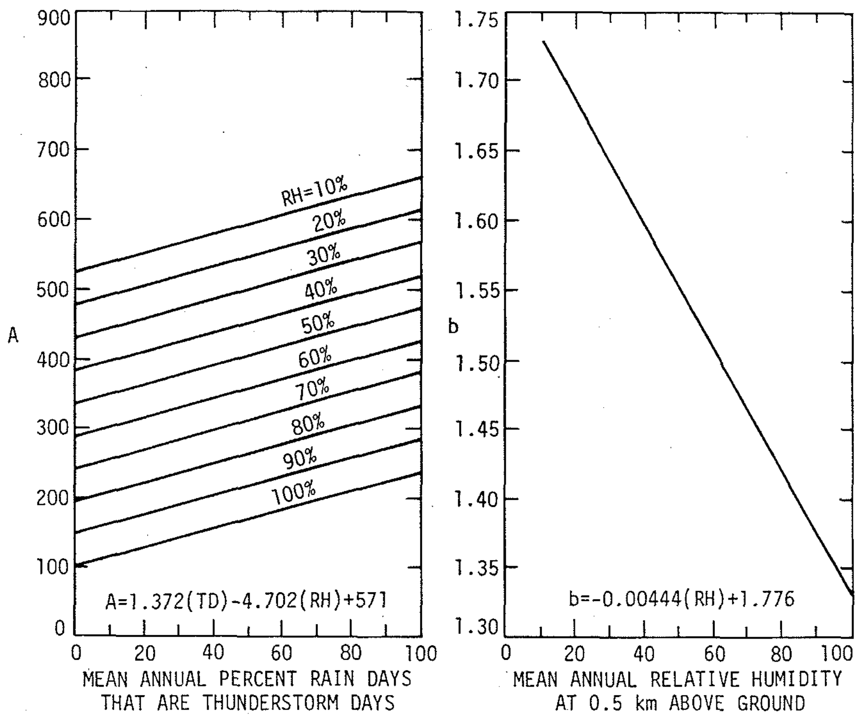

FIG. 1. Graphical method for determination of coefficient and exponent of R in $Z-R$ equation.

used for the nine locations were obtained from nearby areas when not available for the actual location.

The coefficients of the predicted relationships are within 19% (<1 db in Z) in eight of the nine cases. The only case outside of this value was Franklin, N. C., where the difference was 44% (<2 db in Z). The predicted relationships can also be examined at a particular rainfall rate. If a rain rate of 20 mm hr^{-1} is chosen, eight of the nine predicted relationships are within 1.8 db in Z of the actual Z-R relationships. North Carolina has a 2.9-db difference, which represents the poorest fit location. A possible cause of the large discrepancy between the predicted and actual Z-R equation in North Carolina may be due to the difference in the mean percentage of rain days that were thunderstorm days, which was used in the prediction, as compared with that

TABLE 2. Comparison of actual and predicted Z-R equations for the nine sampled locations.

Location	Mean annual per cent of rain days that are thunderstorm days	Mean annual relative humidity at 0.5 km above ground	Z-R equation Actual	Z-R equation Predicted
Miami, Fla.	55	75	$Z=286R^{1.43}$	$Z=293R^{1.44}$
Island Beach, N. J.	20	66	$Z=256R^{1.41}$	$Z=288R^{1.48}$
Franklin, N. C.	41	62	$Z=234R^{1.39}$	$Z=337R^{1.50}$
Champaign, Ill.	43	64	$Z=372R^{1.47}$	$Z=328R^{1.49}$
Corvallis, Ore.	4	55	$Z=301R^{1.64}$	$Z=318R^{1.53}$
Woody Island, Alaska	0.5	73	$Z=267R^{1.54}$	$Z=227R^{1.45}$
Majuro, Marshall Islands	6	85	$Z=221R^{1.32}$	$Z=180R^{1.40}$
Bogor, Indonesia	100	85	$Z=305R^{1.44}$	$Z=307R^{1.40}$
Flagstaff, Ariz.	90	30	$Z=593R^{1.61}$	$Z=553R^{1.64}$

TABLE 3. Comparison of existing and predicted Z-R equations for five sampled locations.

Location	Mean annual per cent of rain days that are thunderstorm days	Mean annual relative humidity at 0.5 km above ground	Z-R equations Existing	Z-R equations Predicted
Karlsruhe, Germany	16	78	$Z=184R^{1.26}$	$Z=225R^{1.43}$
Entebbe, Br. E. Africa	100	73	$Z=278R^{1.30}$	$Z=360R^{1.45}$
Tucson, Ariz.	74	38	$Z=520R^{1.81}$	$Z=494R^{1.61}$
Locarno, Switzerland	42	65	$Z=300R^{1.50}$	$Z=320R^{1.49}$
Cristobal, Panama	34	80	$Z=308R^{1.36}$	$Z=240R^{1.42}$
Montreal, Canada	22	68	—	$Z=281R^{1.47}$

TABLE 4. Statistical analysis of differences between existing and predicted equations for the five independent locations.

Location	Difference in A between existing and predicted equations	Per cent difference in A	Difference in b	Per cent difference in b	db difference at $R=1$ mm hr^{-1}	db difference at $R=5$ mm hr^{-1}	db difference at $R=10$ mm hr^{-1}
Karlsruhe, Germany	41	22	0.17	14	0.9	2.1	2.6
Entebbe, Br. E. Africa	82	29	0.15	12	1.1	2.1	2.6
Tucson, Ariz.	26	5	0.20	11	0.2	1.6	2.2
Locarno, Switzerland	20	7	0.01	0.7	0.3	0.9	0.2
Cristobal, Panama	68	22	0.06	4	1.1	0.8	0.5
Average	47.7	17.0	0.118	8.3	0.7	1.5	1.6

which actually occurred during the data collection period (41% for the former and 36% for the latter).

The comparison in Table 2 demonstrates the relative accuracy of the method, since it is applied to the data from which it was derived. However, the method must be applied to independent data to test it fully. This was done in Table 3. The first two existing Z-R equations were derived from drop spectra determined by Diem (1966) with the filter paper technique. The third was determined by Foote (1966) also with the filter paper method. The Locarno relationship was determined with a raindrop spectrometer (Joss and Waldvogel, 1967). The fifth equation was obtained with the raindrop camera in the Panama Canal Zone. The final equation is a prediction for the indicated location; no relationship has been established for this area as yet.

A statistical analysis was performed on the five locations listed in Table 3, and the results are presented in Table 4. The average differences in the A and b values between the existing and predicted equations are 47.7 and 0.118, respectively, which compare well with the S.E. of A and b (Table 1) for the original nine equations. The db differences between the actual and predicted equations at various rainfall rates are also indicated.

4. Evaluation

Since the object of the above study was to yield some improvement in the estimation of rainfall amounts with radar, an evaluation of the technique is necessary. To be of practical value, it should allow for a more accurate determination of rainfall amounts with radar for individual storms when compared to other means of choosing Z-R equations. In Table 5, 14 storms of at least 30-min duration occurring at Champaign, Ill., are evaluated using the Marshall-Palmer equation and the predicted relationship for Champaign (Table 2). The Marshall-Palmer equation was chosen because it is often used for an area when a Z-R relationship has not been determined from drop-size data. For each storm, the equations are compared at various rainfall rates with the actual Z-R equation determined from drop-size data collected at that location. For $R = 1$ mm hr^{-1}, the average db difference in Z for the 14 storms combined

TABLE 5. Evaluation of 14 Illinois storms using the Marshall-Palmer and predicted equation for various rainfall rates.

Storm date	Number of m^3 samples collected	Actual Z-R equation for storm	db difference in Z using Marshall-Palmer equation at $R=1$ mm hr^{-1}	$R=5$ mm hr^{-1}	$R=10$ mm hr^{-1}	db difference in Z using Illinois equation at $R=1$ mm hr^{-1}	$R=5$ mm hr^{-1}	$R=10$ mm hr^{-1}
7-16-53	45	$Z=313R^{1.46}$	1.5	0.6	0.1	0.2	0.3	0.4
10-26-53	40	$Z=179R^{1.01}$	0.9	5.0	6.8	2.6	5.9	7.3
10-27-53	100	$Z=202R^{1.50}$	0.4	1.1	1.4	2.1	2.0	1.9
11-20-53	40	$Z=391R^{1.40}$	2.5	1.1	0.5	0.8	0.2	0.04
5-31-54	34	$Z=374R^{1.35}$	2.3	0.6	0.2	0.6	0.3	0.7
7- 2-54	30	$Z=395R^{1.33}$	2.5	0.7	0.2	0.8	0.2	0.7
7-20-54	47	$Z=439R^{1.27}$	3.0	0.7	0.3	1.3	0.2	0.8
7-21-54	44	$Z=550R^{1.28}$	4.0	1.7	0.8	2.2	0.8	0.2
8- 8-54	35	$Z=218R^{1.21}$	0.04	2.8	3.9	1.8	3.7	4.5
8- 9-54	56	$Z=418R^{1.47}$	2.8	1.9	1.5	1.1	1.0	1.0
8-18-54	38	$Z=429R^{1.43}$	2.9	1.7	1.2	1.2	0.8	0.7
8-19-54	59	$Z=413R^{1.17}$	2.7	0.3	1.6	1.0	1.2	2.1
9-19-54	127	$Z=457R^{1.35}$	3.2	1.4	0.7	1.4	0.5	0.1
10-10-54	53	$Z=475R^{1.38}$	3.3	1.8	1.1	1.6	0.9	0.6
Average	—	—	2.3	1.5	1.5	1.3	1.3	1.5

between the Marshall-Palmer and the actual Z-R equation is 77% greater than that for the predicted and actual relationships. At $R=5$ mm hr^{-1}, this difference is 15%. There is no significant difference for $R=10$ mm hr^{-1}.

5. Conclusion

A model has been introduced which allows an estimation of the coefficient and exponent in the Z-R equation, based on the mean annual per cent of rain days that are thunderstorm days and the mean annual relative humidity at 0.5 km above ground. The resulting Z-R relationship may then be applied to individual rainstorms for an estimate of rainfall amounts.

Of course, there may be a large variation in the actual equation from storm to storm, and a single predicted Z-R relationship certainly will not be adequate for all storms. However, as was shown in Section 4, the predicted equation fares better than the often used Marshall-Palmer relationship when several storms are examined. Use of the model then does appear to yield some improvement in the estimation of rainfall amounts with radar.

Acknowledgments. This work was supported by U. S. Army Contract DA 28-043 AMC-02071(E) to the Illinois State Water Survey at the University of Illinois, Urbana. I would like to thank Mr. G. E. Stout, Dr. E. A. Mueller, Mr. A. L. Sims and Mr. S. A. Changnon, Jr., for their helpful suggestions and discussions.

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