Raindrop-Size Distributions in Humid Continental Climates, and Associated Rainfall Rate-Radar Reflectivity Relationships

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ABSTRACT

Raindrop-size spectra obtained with the raindrop camera have been analyzed from two locations, Island Beach, N. J., and Franklin, N. C. The spectra were analyzed with respect to total number of drops per average rain rate per cubic meter of sample, geometric mean diameter, mode diameter, and the diameter of drops at which half the liquid water content lies above that diameter and half below. The results indicate that the distributions from both locations are quite similar for corresponding rainfall rates. Rainfall rate-radar reflectivity relationships indicate that cold frontal rains in these areas generally have smaller drops than warm frontal rains. In addition, it was found that upslope rains are composed of smaller drops than rains of similar synoptic conditions without upslope effects. Finally, a small sampling of a tropical storm rain revealed that small drops may be characteristic of this type of rain.

1. Introduction

Raindrop-size distributions have been analyzed for Island Beach, N. J., and the Coweeta Hydrologic Laboratory near Franklin, N. C., two locations where data were collected with the raindrop camera, a device that photographs raindrops as they fall, as described by Jones and Dean (1953) and Mueller (1960). Island Beach, located on the Atlantic coast, at 39°52'N, 74°05'W, has a humid continental climate modified to some degree by the ocean, while Coweeta, located in southwestern North Carolina, at 35°02'N, 83°28'W, approximately 225 mi from the ocean, has an unmodified humid continental climate. This type of climate, according to Köppen's classification (Haurwitz and Austin, 1944), is a warm, temperate, rainy climate without a dry season and with a warm summer. The actual location of the raindrop camera at Coweeta was at an elevation of 4450 ft MSL in the Nantahala Mountains, approximately 1 mi SE of a mountain peak which is 5000 ft MSL in elevation. Therefore, when the wind is from the southeast, upslope effects play a role in precipitation over the area. Related data from Miami, Fla., were also used in the analysis.

It is hoped that knowledge of the types of drop-size distributions that exist in these regions will improve existing relationships between rainfall rate R and radar reflectivity factor Z, and aid in developing a method whereby rainfall amounts for storms could be measured remotely by radar. Since radar reflectivity is directly dependent on drop-size diameter, one must know the drop-size distributions. The accuracy of radar measurements of rainfall amounts is limited greatly by the apparent lack of consistency in the drop-size distributions for various meteorological conditions. Even when these conditions appear to be nearly the same, the drop-size distributions do not remain constant.

The approach taken in this study was to stratify the data to include only those distributions that are the same or nearly so. Then, for example, if drop-size distributions varied according to the synoptic condition producing the rain, separating or stratifying the drop data according to the synoptic condition would result in nearly constant drop-size spectra. This would mean that the scatter of points around a rainfall rate-radar reflectivity regression line would be reduced. Consequently, a measure of rainfall rate from a known radar reflectivity would be more accurate. Stratifying the data according to rain type, synoptic type, or degrees of stability associated with the precipitation has been found to be appropriate, the last two being the most effective in separating the data into nearly constant drop-size distributions.

2. Spectra analysis

Data used in the analysis were collected with the raindrop camera which takes seven pictures, approximately 1 sec apart, at the beginning of a minute, and then becomes inactive for the remainder of the minute. Each frame represents a volume of about 1 m³ so 1 min of data represents approximately 1 m³. The drops are measured individually, and their number and size, along with other parameters, are punched onto data cards (Mueller and Sims, 1967a,b).

In order to find general trends and characteristics associated with the distributions, it was necessary to examine average drop-size spectra at each location,
rather than individual minutes of data. The averages were determined by sorting the data from each location in ascending order according to rainfall rate, and then grouping them into intervals 1.0 mm hr⁻¹ wide at the lowest rates, the interval increasing in size at higher rates as the number of samples decreased. Rainfall rates were determined from the drop-size distributions. The average number of drops per cubic meter in each 0.1-mm increment of drop diameter from 0.5-7.9 mm, along with other related parameters, such as median volume diameter, geometric mean diameter, mode, and average rainfall rate, was calculated by a computer for each interval of rainfall rate. For this part of the analysis, all of the data were grouped together without stratification.

Upon examining the average distributions from both areas, it was found that they are quite similar. For low \( N_p \)'s, where \( N_T \) is the total number of drops per cubic meter for a particular rain rate, Coweeta has greater numbers of drops per cubic meter than Island Beach for the same average rain rate, while at high \( N_p \)'s, the reverse is true. In the intermediate \( N_T \) range, both locations have approximately the same drop concentrations for the same average rain rate.

Also examined was the relationship of \( R \) to certain diameters of the drop-size distributions, namely, \( D_0 \), \( D_L \), and \( D_M \), where \( D_0 \) is the geometric mean diameter of the spectra with

\[
\ln D_0 = \frac{1}{N_T} \sum_{n=1}^{N_T} n \ln D_n, \tag{1}
\]

as described in an equation for a log normal distribution fit for the drop-size spectra, \( D_0 \), the median volume diameter, is the diameter of the drop-size where half of the liquid water content of the distribution, for the 1-min sample, lies above that drop-size and half below; and \( D_L \) is the diameter of the drop-size that occurs in the largest numbers. Figs. 1a-c show some representative drop-size distributions for low, moderate and high rainfall rates. In Fig. 1, \( N_T \) is the number of 1-min samples used for each rate. Plots of \( R \) vs \( D_0, D_L \) and \( D_M \) also reveal (as in Fig. 1) the similarity of the spectra at both locations.

3. Rainfall rate-radar reflectivity relationships

The radar reflectivity factor \( Z \) from a single raindrop depends directly on the sixth power of the drop diameter \( D \); therefore, the total reflectivity factor from any rain echo will be

\[
Z = \sum_{D=0.5}^{D=7.9} n_D D^6, \tag{2}
\]

where \( n_D \) is the number of drops of diameter \( D \).

The spread of \( D \) from 0.5-7.9 mm best describes the drop-size interval that was found in the data collection; very few raindrops were larger than 7.9 mm. Drops <0.5 mm were sometimes present; however, the limited accuracy of measurement in this range precluded their use in the distribution.

The curve from Z-R values for each minute of data plotted on log-log coordinates would be a straight line with no scatter of points around the line, if all the rains had the same distribution of drops with only \( N_T \) changing, and the relative percentage of each size remaining the same for different rates. (There are other situations
that would also result in a regression line with no scatter. Unfortunately, this does not occur, so the result is a regression line with a great deal of scatter of points around the line. Unless this scatter is reduced appreciably, a rainfall rate or total rainfall amount evaluation may be overestimated by as much as 46%, and underestimated by 31%, sixty-eight percent of the time. When the Island Beach data are stratified according to PASI and synoptic type, the error in estimating R may be 30% low to 43% high for the former, and 29% low to 42% high for the latter.

Since it is not immediately obvious from Tables 1 and 2 that synoptic stratification reduced the S.E., a discussion of this point is necessary. A statistical analysis performed on the New Jersey data indicated that the synoptic classification scheme was effective in reducing the standard error of estimate.

Chi-square tests for equality of variances of log R about their respective means, and for equality of variances about the regression lines, suggested quite strongly that both of these variances sets were heterogeneous. Assuming heterogeneous variances, the T-test was used to look for significant differences between all comparisons of the log R means, and between regression line slopes. There were numerous statistically significant differences among both log R means and regression slopes. Approximately 80% of the T-values for the 45° regression line comparisons exceeded the 0.05 probability level of significance.

If synoptic classification succeeded in reducing the error variance, it would be expected that a pooled estimate of the error variance computed by weighting the individual error variances of the synoptic classifications, according to their degrees of freedom, would be somewhat less than that determined by a regression of log R vs log Z, for all the data combined as one sample.
The evidence of significant differences between regression lines would suggest that the pooled estimate of error variance would be significantly less. An $F$-ratio of $0.02672/0.02385=1.12$ for the combined error variance over the pooled variance with 3122 and 3037 degrees of freedom, respectively, was greater than that required for significance at the 0.01 probability level. The numerous significant differences between regressions, and the significant $F$-ratio are evidence that the synoptic classification provided a statistically significant reduction in the error variance.

The regression lines, when synoptic stratification was employed, reveal some unexpected results regarding the drop-size distributions in warm frontal and cold frontal rains. First, it should be considered that within each of the above frontal patterns, there are variations; for example, there are slow and fast moving cold fronts resulting in predominantly stratiform type clouds in the former and cumuliform clouds in the latter. Thus, the meteorological conditions such as wind shear, convection or evaporation, associated with the warm and cold fronts, do not necessarily remain constant.

The values for $A$ and $b$ for the warm and cold fronts, as well as the number of samples available from each location, are listed in Table 3. Related information for Miami, another area where raindrop data were collected,
Table 3. Cold and warm front regression parameters.

<table>
<thead>
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<th>Location</th>
<th>Cold front</th>
<th>Warm front</th>
<th>Cold front</th>
<th>Warm front</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>( a )</td>
<td>( b )</td>
<td>( N_s )</td>
<td>( N_s )</td>
</tr>
<tr>
<td>Coweeta</td>
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<td>346</td>
<td>339</td>
</tr>
<tr>
<td>Island Beach</td>
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<td>362</td>
<td>417</td>
</tr>
<tr>
<td>Miami</td>
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<td>187</td>
<td>197</td>
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<tr>
<td></td>
<td>403</td>
<td>1.24</td>
<td>341</td>
<td>341</td>
</tr>
</tbody>
</table>

is also presented. These data show that the warm frontal rains for all three locations have larger coefficients than the cold frontal rains. Atlas and Chmela (1957), however, found that heavy showers and large coefficients in the \( Z-R \) equation occurred more frequently with cold fronts. \( Z-R \) plots of data from Island Beach and Coweeta reveal that 4\% and 15\% of the warm and cold frontal rains, respectively, at Island Beach were composed of rainfall rates \( > 10 \) mm hr\(^{-1} \), while at Coweeta, the corresponding figures were 17\% and 12\% for the warm and cold frontal data.

Fig. 2 shows some comparisons of representative drop-size spectra for warm and cold front rains of similar rainfall rates from Island Beach and Coweeta. In each case, the warm frontal rain shows a greater number of the large size drops, in the range of 1.5 mm diameter and greater, in 3 out of the 4 cases. The results are particularly surprising since the number of samples from each area was relatively large and the data were obtained from several storms in each area. These data indicate that cold front rains, at least in the areas where the data were obtained, generally consist of smaller drops than warm frontal rains of similar rates. However, at Miami when \( R > 7 \) mm hr\(^{-1} \) (Fig. 3), the cold front regression line begins to show greater \( Z \) values for given \( R \)'s than the warm front line, indicating the presence of larger drops for similar rainfall rates. Coalescence on the other hand, according to the above reference, would tend to have a decreasing effect on the regression coefficient of the original distribution because of the larger importance of the smaller drops. Also, in initial warm frontal precipitation, where all of the rain originates in the warm air above the front, evaporation is high as the drops fall into the cooler, drier air below the front, resulting in spectra weighted towards relatively large drops.

As the warm front becomes more developed, the evaporating rain forms clouds below the front, increasing the moisture content, and evaporation of the subsequent falling drops is considerably reduced. Along with this, drizzle and fog often exist in well-developed warm frontal situations where small drops are superimposed on the precipitation formed in the frontal zone aloft. As a result, the overall effect on drop-size spectra in well-developed warm fronts is an increase in the number of small drops reaching the ground.

Then, assuming that coalescence, evaporation and wind sorting all occur in varying degrees, and that nature begins with the same drop-size distribution in the clouds, we may postulate that evaporation and wind sorting are more pronounced in warm frontal rains than in rains associated with cold fronts in the two areas analyzed. Indirectly, this is also saying that warm
fronts in these areas generally are not associated with low-level clouds and drizzle, or that cold front rains are characterized by little raindrop evaporation and a great deal of coalescence during rain formation when compared with warm fronts. The second explanation appears to be a more plausible one at this time.

4. Drop spectra in upslope rains and tropical storms

The location of the raindrop camera at Coweeta was quite favorable for upslope precipitation when a south-easterly wind occurred over the area. While orographic precipitation is that produced by the lifting of moist air over high ground, such as a mountain range, it is not always limited to the ascending ground, but may extend for some distance windward of the base of the mountains (upwind effect), and for a short distance to the lee of the mountains (spillover). Since the raindrop camera at Coweeta was located in a mountainous area, it must be assumed that all of the rains were orographically influenced in some way. In the cases discussed here, upslope rains refer only to those produced by the lifting of moist air over ascending ground, the drop camera being on the windward side in these cases.

Some 450 min of data involving upslope rains were obtained. Data at this location were available from synoptic classifications of the same type with and without upslope effects which allowed for a comparison between the two. The associated Z-R relationships are shown in Table 4. In 4 out of 5 cases, the synoptic classification involving the upslope rains had smaller coefficients than the same classification without the upslope effects. This indicates that the drops are generally smaller for the upslope cases for the same or similar rainfall rates. Work done by Weaver (1966) agrees with this conclusion.

On 19 September 1961, tropical storm Esther, the remnants of Hurricane Esther, passed well offshore from the raindrop camera at Island Beach, causing some light precipitation as its outer fringes brushed the coast. A very limited amount of data was recorded by the camera (33 min). However, the results are of interest. The Z-R equation for this case is

\[ Z = 205 R^{1.43}. \] (4)

The scatter around the regression line is noticeably more for this case than for the other synoptic stratifications at Island Beach. The S.E. is 0.236 as compared with 0.179 for the group with the highest S.E. of the remaining classifications (Table 2); the small number of samples is partially responsible for this. Assuming that the points are normally distributed about the regression line, a S.E. of 0.179 would mean a possible error of ±50% or less in measuring R, whereas a S.E. of 0.236 would mean ±70% or less error in R. The S.E. would have to be reduced to 0.040 to lower the possible error to less than ±10% in determining the rainfall rate from a known Z value. It appears that tropical storms may
indeed contain small drops when compared with other synoptic types. Data from cold and warm front rains and from the tropical storm for similar rainfall rates are compared in Fig. 4. It indicates that the tropical storm is composed of many small drops and few large ones when compared with the other distributions.

5. Summary

Results indicate that drop-size distributions at Island Beach, N. J., and Franklin, N. C., both of humid continental climates, are quite similar for corresponding rainfall rates. It was also found that for these areas the best means of stratifying the data for rainfall rate-radar reflectivity relationships is according to synoptic type. Cold front rains were composed of smaller drops than warm front rains for similar rainfall rates, which was unexpected. Upslope rains in North Carolina contained smaller drops than similar synoptic conditions without upslope effects. A small sampling of tropical storm rain obtained in New Jersey revealed the existence of relatively small drops when compared with other spectra.

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REFERENCES


