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THE PERFORMANCE OF DIFFERENT LOSS MODELS IN THE SIMULATION OF STREAMFLOW

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1. INTRODUCTION

In Jakeman *et al.* (1990) and Jakeman and Hornberger (1993), a new approach has been proposed to separate hydrographs and parametrize their response behaviour by constructing linear transfer function models for describing the rainfall-runoff process. An important step in this approach is that the measured rainfall be modulated by a non-linear loss model, to be transformed to excess rainfall, before it is input to the linear transfer function model. In this paper, the performance of three different formulations of the loss model are investigated by examining rainfall, surface air temperature and runoff data from two catchments, one in the USA and the other in Australia.

2. THE MODEL

The model (Jakeman *et al.* 1990; Jakeman and Hornberger 1993) consists of two modules, one non-linear and the other linear. The non-linear module represents the rainfall-loss processes, or the transformation of measured rainfall r_k to excess rainfall u_k. At each time step k a catchment wetness index, s_k is calculated by a weighting of the rainfall time series, the weights decaying exponentially backward in time from step k, namely

$$s_k = cr_k + (1 - \tau_w^{-1})s_{k-1}, \tag{1}$$

where τ_w and c are parameters. The excess rainfall is computed using

$$u_k = s_k r_k. \tag{2}$$

The parameter τ_w in (1) is a time constant, or inversely, the rate at which the catchment wetness

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declines in the absence of rainfall. The parameter c is chosen so that the volume of excess rainfall u_k is equal to the total streamflow volume over the calibration period.

The linear module converts excess rainfall u_k at time step k to streamflow y_k. It has quick and slow flow components of the form

$$\begin{cases} x_k^q = -\alpha_q x_{k-1}^q + \beta_q u_k \\ x_k = -\alpha_s x_{k-1} + \beta_s u_k \\ y_k = x_k^q + x_k + \xi_k, \end{cases} \tag{3}$$

where α_q, α_s, β_q and β_s are parameters and ξ_k represents the addition of all data and model errors.

The parameter α_q (α_s) represents the rate of decay, or equivalently the time constant τ_q (τ_s), of the quick (slow) flow hydrograph following a unit input of rainfall excess; $\tau_q = -\Delta/\ln(-\alpha_q)$, $\tau_s = -\Delta/\ln(-\alpha_s)$, where A is the time sampling interval. The parameter $\beta_q(\beta_s)$ defines the peak of the quick (slow) component of the unit hydrograph. The volume $v_q(v_s)$ of water passing through the quick (slow) component is a function of α_q and β_q (α_s and β_s); $v_q = \beta_q/(1+\alpha_q)$, $v_s = \beta_s/(1+\alpha_s)$. The parameters τ_q, τ_s, v_q and v_s, are known as dynamic response characteristics for a given catchment, as are the loss model parameters τ_w, c and f given below in (4)-(6).

3. LOSS MODEL MODULATIONS

To account for various physical processes incurred in a catchment, equations (1) and (2) of the loss model are modulated in three different ways as presented in the following.

Case 1

A simple function of temperature is used to modulate the observed rainfall. Then r_k in (1) and (2) is replaced with the adjusted rainfall (Jakeman *et al.* 1990)

$$r_k^*(t_k) = r_k(1 - t_k f^{-1}) \tag{4}$$

where t_k is the temperature in degree Celsius at time step k, and f is a modulation factor, which determines how the measured rainfall is modulated with temperature.

Case 2

To account for fluctuations in evapotranspiration and losses to stream, a function of temperature is used to modulate the rate at which the catchment dries out. This is a more physically plausible concept than that in Case 1. Then τ_w in (1) is replaced with

$$\tau_w^*(t_k) = \tau_w \exp[(20 - t_k)f] \tag{5}$$

where f is a modulation factor, which determines how $\tau_w^*(t_k)$ varies with temperature.

Case 3

Apart from the modulation described in Case 2 above, it is supposed that the excess rainfall at time step k goes into stream immediately after arriving at the catchment store, and it is therefore taken out of the storage. This allows temperature to modulate evapotranspiration only. Then (1) is replaced with

$$Sk = c(r_k - u_k) + [1 - \tau_w^{*-1}(t_k)]s_{k-1}. \tag{6}$$

4. DATA DESCRIPTIONS

Two catchments are selected to perform model estimation and simulation. One is the Coweeta Watershed 36 (C36), a 0.486 km^2 catchment in North Carolina. The other is Queanbeyan River (QBN), a 490 km^2 catchment near Canberra. The periods of daily rainfall, streamflow and temperature records used for analysis are 1943-1962 for C36 and 1973-1990 for QBN. For C36, models are estimated for 17 overlapping periods of three water years (low flows on which to commence the analysis can generally be found close to the beginning of a water year), the starting point for each period advancing a year at a time. Among the 17 periods, 6 are non-overlapping. For QBN, models are estimated for the 5 periods selected in Schreider *et al.* (1993), which are practically non-overlapping, and each of which is of approximately two years duration. The starting dates for each of the non-overlapping periods for both catchments are given in Table I.

As an indicator of hydroclimatology, the catchment yields (percentage of runoff to rainfall) for each of the analysed periods are also given in Table I. As seen from the table, C36 is a relatively humid catchment, with yield varying from 77 per cent to 84 per cent, whereas QBN is much drier, with yield from 16 per cent to 34 per cent.

5. THE MODEL PERFORMANCE

The coefficient of determination D, which is the fraction of variance of streamflow explained, is used as an indicator of model performance. For the 6 non-overlapping C36 and the 5 QBN data periods, estimation and simulation performance on independent periods are investigated. Along with the 6(5) values of D from the estimated models, 30(20) D values are obtained by simulating each model on the remaining 5(4) independent data sets. Table II lists the 6(5) D values from the estimation models (bold) and the 30(20) D values from the simulation models.

As shown in the table, for C36 the D values for both model estimation and model simulation are, in general, quite high for all the three cases. The mean values (and standard deviations) of the underlined D for the three cases are, respectively, 0·85 (0·05), 0·86 (0·04) and 0·82 (0·05), while the mean values of all the remaining D are, respectively, 0·82 (0·05), 0·83 (0·04) and 0·78 (0·05). For both estimation and simulation periods, Case 2 yields the best performance, although not

Table I. Starting dates and catchment yields of the analysed data periods

	Coweeta Watershed 36						Queanbeyan River				
Period Number	1	2	3	4	5	6	1	2	3	4	5
Starting Date	Oct 1943	Oct 1946	Oct 1949	Oct 1952	Sept 1955	Oct 1958	Feb 1973	Jan 1975	Oct 1976	Apr 1983	Jan 1988
Yield (%)	77	83	84	77	78	79	26	33	28	16	34

Table **II**. Coefficients of determination *D* for estimation (bold) and simulation periods

		Coweeta Watershed 36 Model number						Queanbeyan River Model number				
	Period number	1	2	3	4	5	6	1	2	3	4	5
Case 1	1	**0·89**	0·81	0·88	0·87	0·88	0·87	**0·91**	0·89	0·88	0·88	0·71
	2	0·77	**0·85**	0·78	0·81	0·80	0·80	0·85	**0·87**	0·87	0·85	0·56
	3	0·80	0·70	**0·80**	0·78	0·79	0·77	0·65	0·65	**0·67**	0·71	0·72
	4	0·86	0·84	0·86	**0·88**	0·86	0·85	0·55	0·57	0·65	**0·67**	0·50
	5	0·89	0·86	0·89	0·88	**0·90**	0·89	0·49	0·50	0·56	0·61	**0·66**
	6	0·75	0·73	0·75	0·75	0·76	**0·77**					
Case 2	1	**0·89**	0·83	0·87	0·87	0·88	0·87	**0·87**	0·86	0·85	0·72	0·80
	2	0·78	**0·86**	0·80	0·83	0·80	0·82	0·84	**0·86**	0·86	0·77	0·82
	3	0·81	0·75	**0·82**	0·80	0·82	0·81	0·70	0·72	**0·73**	0·72	0·71
	4	0·86	0·84	0·82	**0·88**	0·85	0·84	0·34	0·50	0·58	**0·73**	0·16
	5	0·90	0·86	0·90	0·88	**0·90**	0·88	0·52	0·60	0·63	0·59	**0·76**
	6	0·78	0·75	0·79	0·77	0·78	**0·81**					
Case 3	1	**0·88**	0·74	0·82	0·86	0·87	0·87	**0·79**	0·77	0·76	0·60	0·72
	2	0·73	**0·75**	0·75	0·75	0·73	0·72	0·81	**0·84**	0·82	0·67	0·79
	3	0·78	0·73	**0·79**	0·79	0·79	0·78	0·65	0·69	**0·72**	0·70	0·71
	4	0·80	0·74	0·77	**0·82**	0·80	0·79	0·00	0·00	0·35	**0·68**	0·00
	5	0·86	0·76	0·82	0·85	**0·86**	0·85	0·54	0·65	0·68	0·57	**0·74**
	6	0·79	0·63	0·75	0·78	0·79	**0·79**					

substantially better than that for Case 1, but the superiority of Case 1 and Case 2 to Case 3 is certainly significant. Visually, the conclusion is supported by the example of the model fits to the observed data shown as the three top diagrams in Figure 1.

For QBN, the mean values (and standard deviations) of the bold *D* for the three cases are, respectively, 0·76 (0·11), 0·79 (0·06) and 0·75 (0·06), again showing that estimation models have the best performance for Case 2. For simulation, however, the conclusions are not so straightforward. Case 1 yields a reasonably good performance, as do Case 2 and Case 3 for periods 1-3, but the latter cases can have poor performance for the remaining periods. From the three bottom diagrams in Figure 1, we can see that the modelled flow is badly overestimated for Case 3, which is apparently the reason for the low *D*.

As an additional performance criterion, we also considered to what extent the model parameters (as DRCs) are independent of climate sequence (represented by catchment yield) in estimation periods (Jakeman *et al.* 1993). Because of the lack of data periods available for QBN, only the DRC plots for C36 are shown here. As seen from the DRC plots in Figure 2, the scatter of points indicates no strong general trend for most of the parameters of the three cases. The parameter τ_s exhibits a possible dependence on yield in all three cases. Case 3 also shows some dependence for *c* and τ_q, while Case 1 does for *c*. Case 2 is visually superior.

6. CONCLUSIONS

Two catchments were selected to perform model estimation and simulation, using a lumped, unit hydrograph model which has two components, one of them being a non-linear rainfall loss model. The performance of three different formulations of the rainfall loss model is investigated.

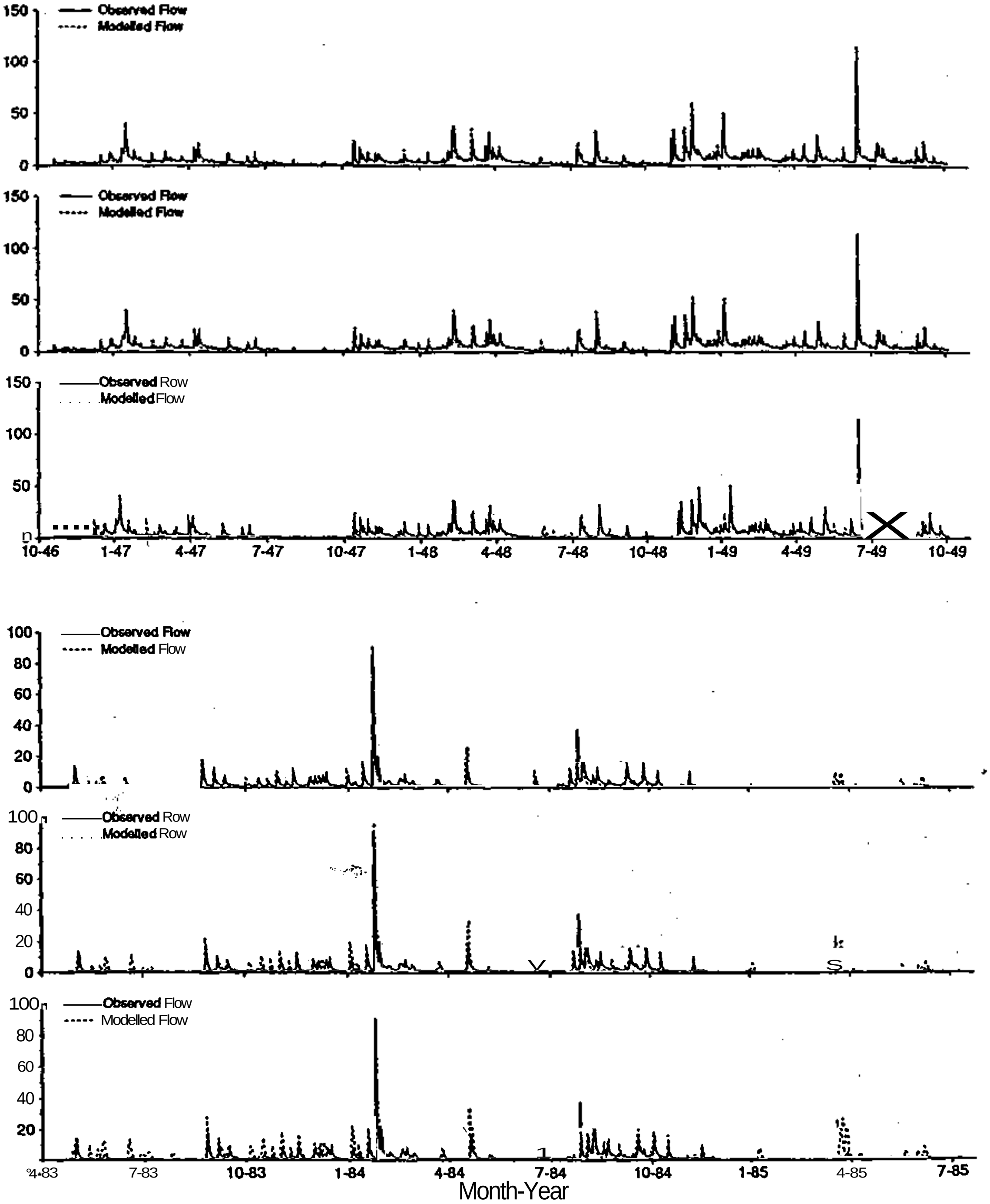

Figure 1. Observed and modelled daily streamflows. Top three graphs, model estimation of period 2 for Coweeta Watershed 36 (mm); bottom three graphs, simulation of model 3 on period 4 for Queanbeyan River (cumecs)

For C36, which is a small humid catchment, the model performs quite well in terms of both estimation and simulation for all of the three cases, but Case 2 is clearly superior. For QBN, which is relatively dry, model estimation performance for all of the three cases is reasonable, and again the best for Case 2. For model simulation Case 1 has reasonably good performance, but for some data periods Case 2 and Case 3 exhibit poor performance.

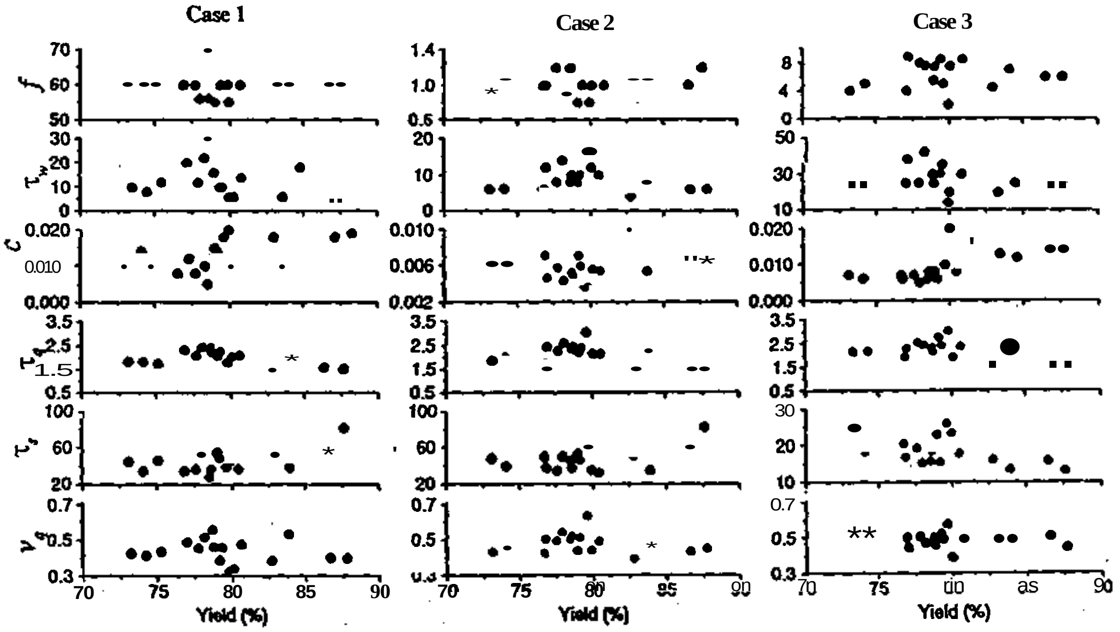

Figure 2. Estimated dynamic response characteristics against yield for the 17 Coweeta (Watershed 36) periods

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