REFINEMENTS IN THE Z-T METHOD OF EXTREME VALUE ANALYSIS FOR SMALL WATERSHEDS

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1. INTRODUCTION

Zelenhasic and Salvai (1987) described the "Z-T" method for determining recurrence intervals for streamflow droughts in a 8,800,000 ha river basin. That report is an extension of the work by Todorovic and Zelenhasic (1970). In this paper we will show that the Z-T method of drought analysis can be applied to a smaller (760 ha) basin if certain refinements are made. For small-sized catchments, one is more likely to find longer records for precipitation than streamflow. An objective of this study was to characterize a specific drought event in the Southern Appalachian Mountains of North Carolina with the Z-T method using both streamflow and long-term precipitation records.

The Z-T method was chosen for this application because it is statistically rigorous and permits estimation of recurrence intervals for both drought duration and cumulative flow deficit. Results are appropriate for generating a synthetic drought record for given recurrence intervals.

The Z-T method is described in detail elsewhere (Zelenhasic and Salvai, 1987). It is briefly outlined here in terms of streamflow as background to the following discussion of refinements in this application appropriate for smaller basin flow and precipitation data. Initially, daily flows (or other data) are ranked for the entire station history in order to identify the reference value (Qr) that caps the lowest 10 percent of all data (i.e., the 90 percent exceedance flow). These become the deficit flow values. The sequential flow record is truncated into alternating periods of flows above and below Qr. Low-flow periods may be punctuated by short events of flow above the reference value, usually caused by storms that briefly raise streamflow but have no long-term effect upon accumulating drought conditions. The next step then is to remove these brief inter-drought periods and to combine adjacent drought periods that effectively function as a single event. Similarly, short low-flow periods that do not function as droughts are censored from the list. The goal is to obtain a list of separate and independent events representing the lowest 10 percent of flows on record. From this, lists of drought duration events and drought magnitude (cumulative deficit below the Qr value) events become the data points for succeeding calculations.

Several tests are applied to ensure that the selected events are independent, identically distributed random variables. First, a chi-square test determines if the distribution of the number of droughts each year fits a time-dependent Poisson process. Then, the lists of both deficits and their durations are tested to ensure they are not serially correlated and do not include runs of consistently increasing or decreasing values. Finally, the correlation between rank numbers for drought deficits and for drought durations is tested to assure that both measures of drought severity rank the same events in essentially the same order. The responsive nature of small streams and precipitation data necessitated refinements in procedures in order to meet these test standards.

The resulting lists of discrete extreme events are fit as cumulative relative frequency distributions of the form $y = 1-\exp(-ax)$, and the goodness of fit is tested by the Kolmogorov-Smirnov test. The parameters of the cumulative distribution functions are, in turn, used to calculate the distribution functions and recurrence intervals for the annual maximum drought deficits and annual maximum durations.

2. APPLICATION

The Z-T method was applied in this study to streamflow and precipitation data to describe the southeastern drought of 1984-86. The analysis is part of the National Science Foundation's Long Term Ecological Research Program study of the effects of this event upon Southern Appalachian and Coastal Plain forest ecosystems (Swift and Blood, 1987). The 52-year record of flow from Watershed 8 at the Coweeta Hydrologic Laboratory in western North Carolina shows two extreme low-flow periods in the 1940's and 1980's, separated by more than 30 years of near- or above-average flows (Figure 1). The longest precipitation record near Coweeta (109 years) is at Highlands, NC. Figure 2 shows several multiyear periods with well-below-average precipitation, most notably a 4-year period in

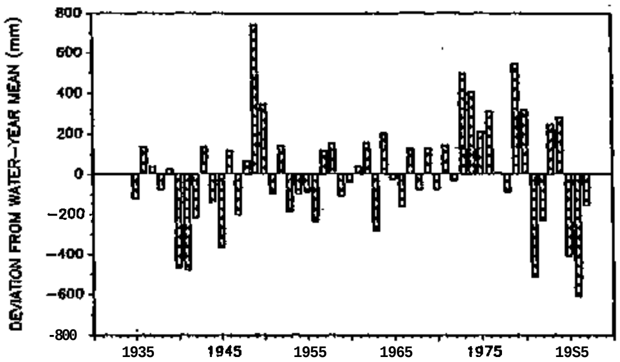

Fig. 1. **Coweeta** Watershed 8 annual **streamflow** expressed as deviations from the 52-year mean water-year flow.

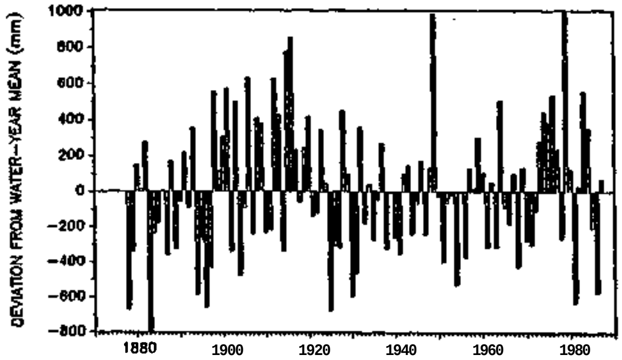

Fig. 2. **Highlands**, NC annual precipitation expressed as deviations from the 109-year mean water-year total.

the 1890's. The question posed is: how unusual was the 1984-86 period compared to other low flow or low rainfall periods in the past 109 years? Table 1 lists the various data sets used in this study. Swift et al. (1988) give further details of the Coweeta **hydrometeorological** network. Watershed 8 (WS08) is a 760-ha catchment, covered with mixed deciduous forest and drained by Shope Fork, a fourth-order headwater tributary of the Little Tennessee River. Raingage 19 (RG19) is at the mouth of the basin and is reported by the National Climatic Data Center as "Coweeta Exp Station." The Highlands gage, presently operated by the Highlands Biological Station, is a station in the US Historical Climatology Network. The Highlands site is 20.4 km east of the Coweeta site at an elevation equivalent to the upper slopes of WS08. This study pursued three questions: 1) can the Z-T method be applied to a small stream such as WS08; 2) can the Z-T method be applied to a precipitation record for the same watershed and does it identify the same extreme events as do flow data; and 3) can a longer precipitation record also identify the same events and how do the present droughts rank in comparison to other events occurring between 50 and 110 years ago?

TABLE 1. Data sets used in Z-T drought analyses.

Site	Data	Elevation (m)	Record length	Observation period day	Observation period month
WS08	Flow	702	1935-1986	X	X
RG19	Precip	686	1935-1986		X
Highlands	Precip	1170	1878-1986		X

Daily flow and precipitation totals were available for WS08 and RG19, whereas Highlands precipitation was readily available only as monthly totals. Although daily flow data were easily used in the Z-T method, RG19 daily precipitation data had to be smoothed as monthly totals.

For daily streamflow, three approaches for defining the lowest 10 percentile reference value, Qr, were tried, each with particular advantages. First, a single, annual Qr was obtained by using all daily values in the year without any stratification by month or season. Because of the seasonality of Coweeta streamflow, this truncation method forces most droughts to occur in the low-flow season and is most appropriate for analyses to identify water-supply problems. In the second approach, a Qr was obtained separately for each month. As a result, the truncation step identified some drought periods in every month providing useful information for ecosystem studies. Although an unusually low flow in a spring month, for example, might still be greater than record low flows in the fall, the impact of much lower than usual spring flows upon the stream ecosystem and organisms living therein could be critical. However, Qr values may be very different for adjacent months, possibly causing the start or end of a dry period to occur at the first of the month even though flows were identical on adjacent days. To avoid this, a third method applied a continuously varying Qr for each date, calculated by fitting a smooth curve through the monthly Qr values.

The procedures for combining adjacent dry or wet periods strongly influenced the results of the various tests for statistical independence. Zelenhasic and Salvai (1987) discussed an approach for eliminating minor droughts before combining adjacent droughts. They did not specify clearly whether these should be single-pass or iterative operations. Best results were obtained in our analysis when the data sets were modified by iterative processes, and by first combining adjacent drought periods separated by minor wet periods. The rules for combining were: 1) the absolute value of each of the adjacent drought deficits had to be greater than the absolute value of the intervening wet period, and 2) the sum of the two adjacent drought deficits had to be larger than three times the value for the wet. If the rule finds the intervening wet period to be minor, then the combined dry period has the accumulated deficit value equal to the algebraic sum of the two adjacent droughts less the value of the intervening wet. The combined duration becomes the sum of the three durations. The alternative of simply purging the intervening wet period out of the record, followed by Zelenhasic and Salvai (1987), was not used. Our logic was an extension of the finer-scale processes which occur within a single day. Flow might fluctuate above or below the Qr level several times but the value accepted is the entire sum for the day. Several passes through the data, testing for relative magnitudes of adjacent dry and wet periods, are necessary because each combination of two adjacent droughts will establish a larger deficit which may then absorb an adjacent wet period that may have been passed over in previous iterations. The philosophy is to assemble, as completely as possible, each major extended dry period, **unfragmented** by short wet periods that did not represent an effective termination of the low flow regime.

The rule for censoring minor droughts from the record is to omit any deficit that is less than one-hundredth of the maximum deficit **determined by** the combining step outlined above. A benefit of doing the step of combining adjacent droughts first is that some minor droughts, which are actually part of an extended dry period, will be included. Also, the resulting maximum deficit is larger, permitting more of the insignificant **low-flow** periods to be **eliminated.**

Recurrence **intervals** estimated by the Z-T analysis were generally quite sensitive to the procedure chosen for fitting an exponential distribution function to the observed cumulative relative frequencies of drought deficit and drought duration. Several approaches were evaluated to improve the fitting step. The simplest approach was a linearized (by **ln-ln transform)** least squares regression analysis, in which the resulting equation was or was not forced through the origin. The more sophisticated approach of nonlinear least squares regression analyses, with various options of unweighted and weighted fits and several iteration schemes, produced closer fits to observed data in the low and middle ranges of the cumulative frequency curve. However, the resulting function often badly underestimated the frequency of large deficits and durations, and hence overestimated the associated recurrence **intervals.** The linearized regression, forced through the origin, provided the most accurate fits in the tail of the distribution, and thus the most reasonable estimates of recurrence intervals for truly extreme events.

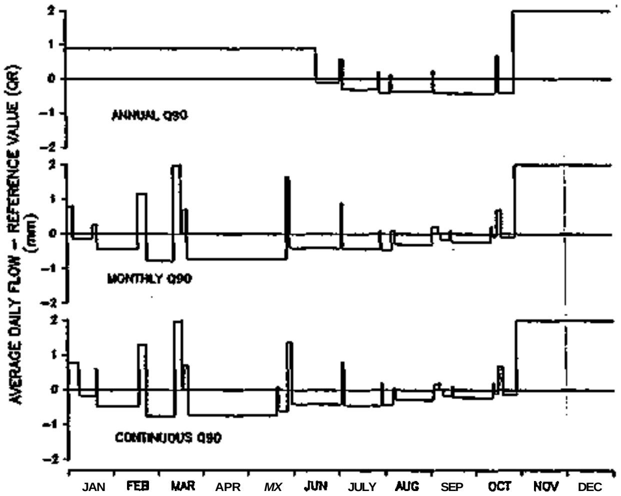

Fig. 3. Average daily flow on Coweeta Watershed 8 in 1986, truncated by a single annual **Qr,** monthly **Qr,** and a continuously varying daily Qr.

3. RESULTS

Figure 3 illustrates differences between the **hydrological** and ecological definitions of drought. The three traces contrast the effect of truncation of 1986 streamflow by three alternate Qr **systems:** one **annual,** 12 monthly, **or** 365 daily Qr values. Applying the same annual Qr value for all days of the year restricts the flow deficit **periods** to summer and fall. This hydrologic definition of drought identified the typical low-flow period of the year when water supply managers depend upon reservoir storage. In contrast, truncation using 12 monthly mean Qr values or 365 daily Qr values defined **ecologically** important drought periods throughout the year. The two more detailed methods yielded essentially the same result: streamflow was unusually low for most of the first 10 months of 1986. In both **cases,** combining adjacent droughts and eliminating minor **interdrought** periods consolidated this part of the record into a single **277-day** drought.

The basic observations of streamflow were either daily totals or monthly totals. The Z-T analysis of daily flows is the most detailed performed for this study. Monthly flow also was analyzed for comparison to monthly precipitation totals. Recurrence intervals for the four largest droughts are shown in Figure 4 for WS08 daily flow totals. The 1986 drought duration was 277 days which has a recurrence interval of 307 years while the 92 mm deficit has an estimated recurrence interval of 233 years (Table 2). The droughts **of 1939,** 1940, and 1941 all rank in the top five **droughts.**

TABLE 2. Comparison of streamflow deficit events determined from daily and from monthly total flow observations at Coweeta Watershed 8.

	Daily total flow			
Year	Duration **(days)**	Recurrence* **(years)**	Deficit (ma)	Recurrence* **(years)**
1986	277	307	92	233
1939	123	15	54	27
1941	147	24	41	13
1981	53	4	24	5
1940	35	3	17	4
1956	37	3	13	3

	Monthly total flow			
Year	Duration (months)	Recurrence* (years)	Deficit (mm)	Recurrence* **(years)**
1986	9	63	135	97
1939	7	32	83	25
1941	5	16	78	22
1985	4	12	36	8
1981	2	6	23	6
1954	3	9	20	5

***Estimated** recurrence interval for drought duration or drought deficit, respectively.

The annual maximum flow **deficits,** based upon analysis of the monthly streamflow totals from WS08, fall generally in the same years as defined by daily flow totals (Figure 5). For

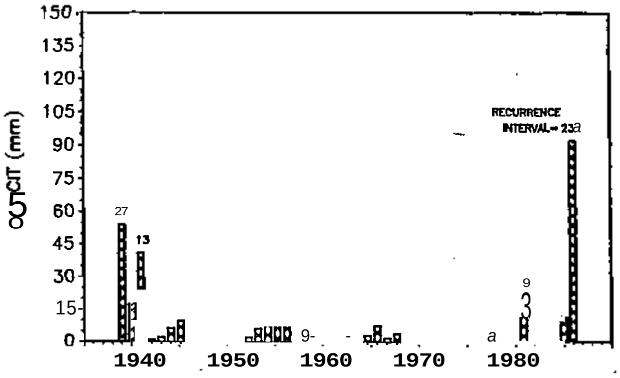

Fig. 4. Annual maximum flow deficits over the 52-year history for Coweeta Watershed 8, based on daily flow data. Significant events are labeled with recurrence intervals (years).

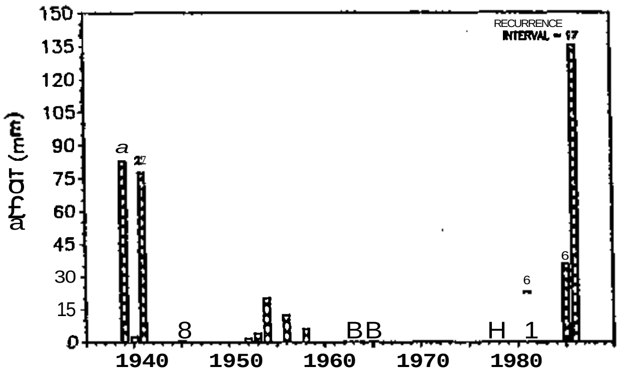

Fig. 5. Same as Figure 4 except based on monthly total flow data.

monthly data, the total flow deficit for 1986 is greater (135 mm), and the recurrence interval is shorter (97 years) than for the analysis based on daily streamflow totals (Table 2). Much of the difference in results between daily and monthly totals is due to masking of short-term events by the smoothing effect of the monthly sum. While a 9-month drought is certainly unusual, 277 consecutive days of low flow is even more unique. The time boundaries of any given month will almost certainly not coincide with daily drought boundaries. For example, daily data for the 1939 drought yielded a duration of 123 days but monthly data indicate a duration of 7 months. At this coarser time scale, the first three months of 1940 were not separate from the dry fall and winter of 1939. Results of the two analyses match closely for drought duration in 1941, and duration, deficit, and recurrence in 1981.

TABLE 3. Comparison of precipitation deficit events determined from Coweeta RG19 and Highlands precipitation gages.

RG19			Highlands		
Year	Deficit (mm)	Recurrence interval (years)	Year	Deficit (mm)	Recurrence interval (years)
1980	60	18	1976	64	28
1964	55	15	1981	61	23
1985	54	14	1980	56	19
1965	48	11	1917	55	17
1986	46	10	1907	53	16
1976	45	9	1925	49	13
1939	38	7	1964	49	13
			1939	33	6
			1986	21	3

Precipitation data did not identify the same drought periods as did streamflow data (Table 3). This study shows that the Z-T procedure can be used with precipitation data and that the requirements of the various statistical tests can be met. However, precipitation data do not describe the same drought phenomena described by streamflow data. Flow responds to individual storm events, but due to its coupling to soil moisture storage, streamflow also responds to the damped effects of all previous storms and droughts. In contrast, precipitation data are entirely event dominated, and an individual storm can terminate a drought period at the low Qr levels used here. Streamflow is a better integrator of drought conditions at the watershed scale than is precipitation.

All drought durations identified by these precipitation data were 2 months or less, and recurrence intervals for precipitation deficits were generally less than intervals defined for flow deficits. Figure 6 shows many more deficits than Figure 5 for the same time period. Precipitation at RG19 ranks highest a pair of 2-month periods in 1980 and 1964, whereas those years are not included in the flow ranking. A

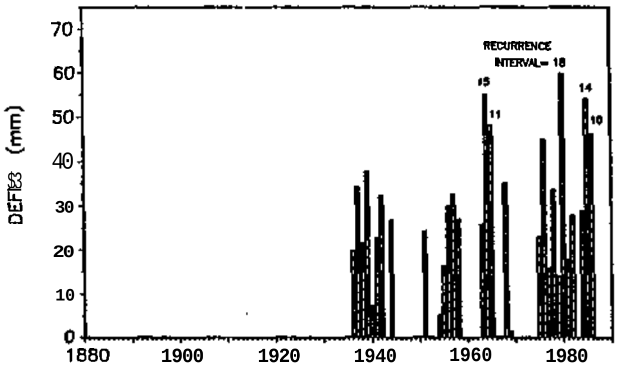

Fig. 6. Annual maximum precipitation deficits over the 52-year history for Coweeta Gage RG19, based on monthly totals.

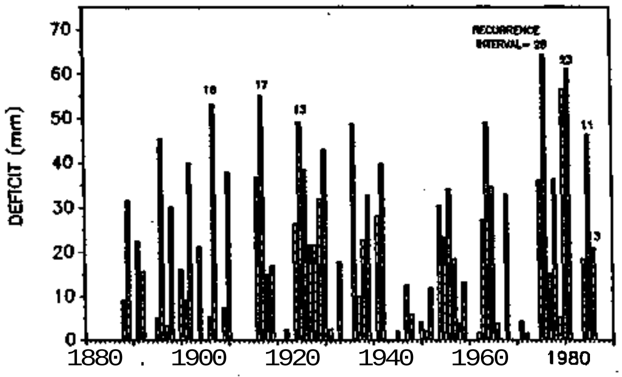

Fig. 7. Same as Figure 6 except precipitation deficits over the 109-year history for Highlands, NC gage.

1985 drought period is ranked by both monthly precipitation and flow at Coweeta, but not by daily flow. The highest ranked droughts in the flow record drop to 5th and 7th in the RG19 analysis.

Of the top 7 annual maximum precipitation droughts identified by RG19 data, 5 were also picked by the longer-duration Highlands data but with some notable changes in sequence. The droughts of 1986 and 1939, which ranked highest in both WS08 analyses were ranked also by precipitation analyses but below the 1964, 1976, and 1980 events which do not even appear in the top 6 streamflow droughts. The third-ranked 1941 streamflow drought does not appear in either precipitation ranking in Table 3. Of the 9 top-ranked precipitation droughts at Highlands, 3 are from the pre-Coweeta period (Figure 7) but the 1890 period is missing from the rankings. The 1925 drought is remembered as the last major drought by long-time residents of the mountains. On the basis of annual precipitation totals from the Highlands record, the 3 to 4 years in the 1890's and the period in 1925-1932 were both expected to place high in drought rankings from the extended history.

4. CONCLUSIONS

Although originally developed for river basin flood and drought analyses, we have demonstrated that the Z-T method can be applied to flow data for small fourth-order streams. Future studies at Coweeta will extend its application to still smaller catchments. The more responsive nature of small mountain watersheds requires that the editing of daily flow data be more rigorous and in a specific sequence relative to procedures previously reported for larger river basins. Also, the analyst should avoid sophisticated analytical routines that weight the fitting of cumulative frequency distributions in favor of midrange values to the exclusion of the more relevant extreme values in the tail of the distribution. For small streams, daily rather than monthly flow totals provide detailed information and thus better estimates of drought deficit and duration and their recurrence intervals. In order to identify seasonally significant low-flow events, a reference value, Qr, should be determined for each flow season or month. A daily varying Qr was not necessary for truncating flow data for this 760 ha watershed. The 10 percentile Qr recommended by Zelenhasic and Salvai (1987) was appropriate for flow data.

As applied here, precipitation data were not satisfactory surrogates for streamflow data in drought analysis. Precipitation records are less representative of drought conditions at the watershed scale because they do not contain information about antecedent soil moisture conditions as do streamflow records. As Figure 2 shows, seasonal or annual precipitation data are likely to carry this needed longer-term information. Another alternative is to use a larger Qr such as the 25 exceedance probability for less sensitive truncation of the precipitation record. Because the Z-T analysis is based upon the distribution of events within the year, it can not be applied to seasonal or longer-term totals. A similar analysis (Sen, 1980) can treat multiyear periods. Our preliminary testing of Sen's method yields closer agreement between annual precipitation and annual flow estimates of drought incidence. The multiyear analysis of precipitation should give a truer ranking to the 1890's drought and enable consideration of the entire current drought period of 1984-1988.

Our analyses were undertaken to characterize the 1984-1986 drought for the Southern Appalachians as part of a larger study on the effects of extended drought on forest and stream ecosystems. Some responses, such as overall forest growth or productivity, may correlate with the hydrologic definition of drought, but occurrences of short-term dry periods during rapid growth in spring and early summer could be even better indices. For terrestrial and aquatic organisms with short life cycles, a drought during a critical reproduction or growth period would disturb their function or possible existence. Thus, for ecosystem studies, the monthly varying reference value (Qr) is required to identify unusually dry periods throughout the year in relation to species life histories and metabolism.

An advantage of the Z-T method is the ability to define the magnitudes and recurrence intervals for both cumulative deficit and drought duration. Current ecosystem studies at Coweeta may differentiate which organisms and processes respond to the duration of the dry period and which to the total deficit of moisture. These studies include patterns of tree mortality and gap-phase succession, measures of tree growth such as leaf area and diameter increment, changes in physiological processes of tree species, shifts in levels of insect defoliation, and restructuring of stream biology. Results have already demonstrated significant changes in precipitation and stream chemistry, and in watershed nutrient budgets, during drought periods. Use of the Z-T method in the overall context of integrated ecosystem research provides a powerful tool for assessing the severity and likely recurrence of hydrologic extremes, as well as impacts of extreme events on ecosystem and population processes.

5. ACKNOWLEDGEMENT

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