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Describing and Testing Nonlinear Treatment Effects in Paired Watershed Experiments

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ABSTRACT. In forest hydrology research, treatment effects are commonly assumed to be proportional to the hydrologic response measured on a control watershed, and are fitted and tested for significance by linear regression techniques. More likely the treatment effect (measured as the difference between pre- and posttreatment regressions of the treatment basin's response on the control basin's response to weather) is nonlinear, unimodal in midrange, and asymptotically zero in one or both extreme ranges. This paper offers simple models for describing such treatment effects and gives illustrative fitting and testing procedures. FOREST SCI. 30:305–313.

ADDITIONAL KEY WORDS. Coastal Plains, hydrology, silviculture, stormflow.

THE HYDROLOGIC RESPONSE to forest operations is traditionally evaluated via paired watershed experiments (Wilm 1944). Two forested watersheds in close proximity to one another and with similar size, topography, vegetation, and soils are selected. Gauging stations are established and surface water discharge is monitored on each during a calibration period. Calibration data may be collected for one or several years prior to the imposition of treatments.

Treatments of interest are then imposed on one of the paired watersheds (Hewlett 1971). The treatments are frequently a sequence of forest operations including harvest and regeneration practices. Portions or all of the treated watershed may be partially or clearcut harvested. Harvested portions may be site prepared and reseeded or replanted to grasses or trees. The question of interest is what is the hydrologic response of similar watersheds to harvest of the mature forest and conversion to alternative land use or reforestation with young trees?

Several responses may be of interest. One is water yield. Harvest of mature trees diminishes evapotranspiration and thus increases runoff. One may wish to quantify the yield increase resulting from reduced evapotranspiration. Reduced transpiration on the treated watershed also typically produces higher soil moisture content during the growing season, implying lower unused moisture storage capacity and thus, greater stormflow discharge and greater peakflow rates from the treated watershed. One may wish to assess these increases and their potential benefit or detriment to downstream interests.

Traditional data analyses consist of establishing a calibration period relationship between the two watersheds using only the pretreatment data. Data from the watershed selected for treatment are considered the response variable(s); data from the control watershed (left undisturbed throughout the experiment) are considered the predictor variable(s). Once a satisfactorily precise predictor of the behavior of the treatment watershed is developed—based on the behavior of the control watershed—treatment is imposed. The data analyst then attempts to determine whether the calibration relationship is altered by the treatment. If the

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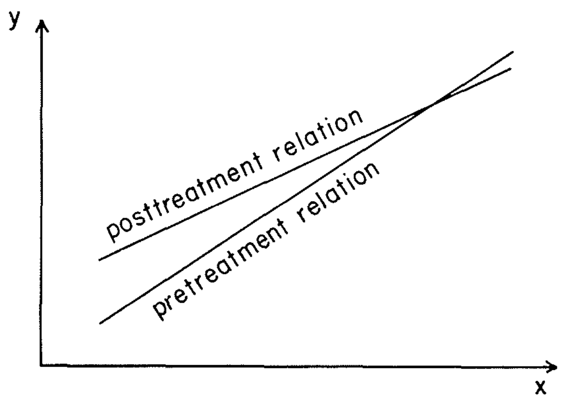

FIGURE 1. Hypothetical pre- and posttreatment linear regressions exhibiting reversal of apparent treatment effect at large responses.

calibration relationship continues to predict the response of the treated watershed as well posttreatment as pretreatment, the treatment is deemed to have produced no significant effect on the hydrologic response of the treated watershed. However, if predictability using the calibration relation deteriorates posttreatment, especially if the predictions become clearly biased, then the bias is attributed to the treatment imposed, and the analyst attempts to partition the posttreatment response as the sum of the predicted response and the treatment effects.

A familiar statistical procedure for describing such changes, and quantifying those that occur, consists of fitting separate regressions pre- and posttreatment—always treating data from the control watershed as the independent variable and data from the treatment watershed as the dependent variable (Wilm 1944, Hewlett 1971). Heretofore, the pre- and posttreatment relations have been considered to be of the *same functional form.* Indeed, it is often found that during the calibration period many responses of interest (for example, monthly water yield, individual storm peakflow rate, or stormflow volume) may be quite well predicted by the simple linear regression model

$$y \triangleq b_0 + b_1x, \tag{1}$$

where y is the treatment watershed response and x is the analogous response on the control watershed. Thus, where a treatment response is detected, one may see published two regressions, say

$$y \triangleq a_0 + a_1x \tag{2a}$$

and

$$y \triangleq b_0 + b_1x, \tag{2b}$$

where (2a) gives the pretreatment relation between, say, water yield on the treatment watershed (y) and the control, and (2b) gives the posttreatment relation between the same two variables. Occasionally the regressions (2a) and (2b) may be fitted to transformations of the original data and/or they may be more complex; for example, they may be quadratic rather than linear and may involve multiple rather than simple linear regression techniques (Hewlett and Helvey 1970, Bryan and Hewlett 1981).

Two problems with this approach are common. Not infrequently, posttreatment regressions fit poorer than pretreatment ones because of both physical and biological reasons. And, too, if linear regressions are fitted for simplicity—and the

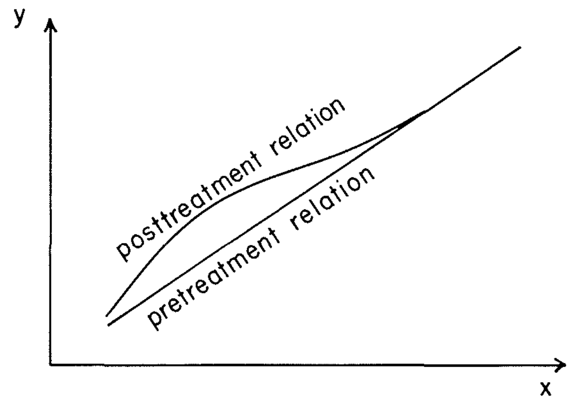

FIGURE 2. Hypothetical treatment effect that is unimodal for midrange responses and asymptotic for extreme responses.

slope coefficients (a_1 and b_1) are allowed to be distinct—the lines (2a) and (2b) may cross near the edge of the data. This implies a reversal in the sign of the treatment effect that often makes little sense hydrologically. Thus, the predictors (2a) and (2b) may appear as in Figure 1, and the reversal in the apparent effect of the treatment at large x is inexplicable—hydrologically—in permeable, well-drained, forested watersheds.

This paper addresses these problems, proposes a form of treatment response that may be widely encountered, gives a very diverse family of models that may be useful in describing such responses, and gives an example of applying the techniques successfully to describe stormflow response to partial clearcutting in a paired watershed study in the coastal flatwoods. It leaves aside introduced problems of fitting and testing models intrinsically nonlinear in the parameters.

ASSUMED FORM OF TREATMENT EFFECTS

We consider treatment effects of the form schematically depicted in Figure 2. Thus, we are content with the considerable empirical evidence that prior to treatment analogous responses on two similar watersheds may be quite simply (even linearly) and strongly related. That is, two nearby watersheds typically receive much the same input from passing climatic events. And, watersheds chosen for similarity in topography, soils, and initial vegetation may respond proportionally to these events. Finally, if they are nearly the same size they may respond nearly identically (so that the pretreatment regression may have an intercept near zero and a slope near one).

However, if the imposed treatment significantly alters the vegetation on the treated watershed (via clearcut harvesting, for example), the same argument cannot be sustained posttreatment. This is reflected in Figure 2 in the fact that the posttreatment relation is of a different functional form. Here we consider altering the functional form of the relation to accommodate treatment effects with the following properties:

1. Effect (if any) is additive.
2. Effect is always positive (or always negative) throughout the range of observed and potential data.
3. Effect is (ordinarily) largest in the midranges of the data and becomes asymptotic to the pretreatment relation at one or both extreme ranges.

We believe that many responses to harvest and regeneration of forests may be examples of responses with the properties 1 through 3. For example, the intercept of the posttreatment regression of stormflow volume should still approach zero as a limit; for clearly, if there is no storm there can be no response from either watershed. If evapotranspiration is less from the treated than from the control watershed, soil water storage opportunity is less; hence, a greater response to rainfall is expected from small to midsized storms. But very large storms overwhelm available soil water storage, implying similar stormflows following very large storms; i.e., the posttreatment response will be asymptotic to the pretreatment relation.

In this example we oversimplified the description of complex hydrologic processes interacting in the forest system. Further, we assumed that hydrologic functions on the treated forest have not been grossly changed; that treatment has hastened or slowed processes such as unsaturated flow, but has not greatly affected flow pathways (i.e., subsurface versus overland). Nonetheless, the next sections give a large family of models for describing treatment effects with the properties 1 through 3, for testing the statistical significance of these effects, and exhibit a successful application.

A Family of Models for Describing Unidirectional, Unimodal Treatment Effects

Consider pretreatment relations of the form

$$y_i \triangleq g(x_i) \tag{3}$$

where $g(\cdot)$ is often the linear function $b_0 + b_1(\cdot)$, and the x_i and/or y_i may or may not be transformed prior to fitting. A large family of models for both pre- and posttreatment data, when the treatment response has the properties 1 through 3 of the previous section, is

$$y_i \triangleq g(x_i) + ktf(x_i), \tag{4}$$

where k is a constant to be determined from the data,
t is an indicator variable (t = zero pretreatment; t = one posttreatment),
and $f(x)$ is the kernal of a continuous, unimodal probability density function.

Notice that by the definition of the variable t, model (4) is

$$y_i = g(x_i) \quad \text{pretreatment; and}$$
$$y_i = g(x_i) + kf(x_i) \quad \text{posttreatment.}$$

That is, model (4) preserves the chosen calibration relation (3) for the pretreatment period, and simply adds a component ($kf(x_i)$) posttreatment to describe the added effect of treatment. The added component is chosen to possess the properties 2 and 3 so that the fitted model necessarily resembles Figure 2.

Some specific examples of the model (4) are as follows:

Example 1. If $g(\cdot) = b_0 + b_1(\cdot)$,
$f(\cdot)$ is kernal of gamma distribution,
then resulting model is

$$y_i \triangleq b_0 + b_1x_i + b_2t\{x_i^{b_3-1}e^{-x_i/b_4}\}. \tag{5}$$

Example 2. If $g(\cdot) = b_0 + b_1(\cdot)$,
$f(\cdot)$ is kernal of two parameter Weibull distribution,
then

$$y_i \triangleq b_0 + b_1x_i + b_2t\{(x_i/b_3)^{b_4-1}e^{(-x_i/b_3)^{b_4}}\}. \tag{6}$$

Example 3. If $g(\cdot) = b_0 {}^*(\cdot)^{b_1} \Rightarrow \ln y_i = b_0 + b_1 \ln x_i$,
$f(\cdot)$ is kernal of log-normal distribution,
then

$$\ln y_i \triangleq b_0 + b_1 \ln x_i + b_2 t \left\{ e^{-\frac{1}{2b_3}(\ln x_i - b_4)^2} \right\}. \tag{7}$$

The models (5), (6), and (7) are each five parameter regression models relating y to x. Each is intrinsically nonlinear in the parameters. In this paper we leave aside computational complexities created by intrinsic nonlinearity, observing only that approximate least squares estimates of the parameters are routinely obtainable by iterative computations—given sufficient data both pre- and posttreatment, and access to appropriate hard and soft computerware allowing nonlinear estimation.

An Illustrative Computation

Columns 1 through 3 of Table 1 give the date of occurrence of 53 individual storms observed in *Pinus elliottii* flatwoods in Bradford County, Florida, from December 1977 to June 1981. Columns 4 and 5 give the measured stormflow volume from a 165-acre treatment and a 345-acre control watershed, WS 1 and WS 3, respectively. Subsequent to a 1-year calibration period (December 1977 to November 1978), 49 percent of WS 1 was clearcut harvested in November and December 1978, chopped in April and August of 1979, bedded in October of 1979, and planted in November 1979 (Swindel and others 1982).

Previous analyses of these data (Swindel and others 1983) suggested that a desirable calibration model was

$$\ln y = b_0 + b_1 \ln x, \tag{8}$$

that the relation (8) was significantly altered at time of harvest, and that there were no further discernable changes in the postharvest period of observation. Here we simply partition the data into pre- and posttreatment periods at time of harvest.

A scatter graph (which the reader may reconstruct by superimposing the data in Figure 3a and 3b) suggested that the model (7) might well describe the pre- and posttreatment data—and hence, the treatment effect. Further, this scatter graph strongly suggested that the treatment effect was largest (about $2 \approx b_2$) at about $\ln x = -4.5 \approx b_4$, and that the treatment effect might have inflection points at about $\ln x = -6.0$ and -3.0. Since b_3, the variance of the appended log-normal kernal, is the square of the distance between the inflection points and maximum effect, we took initial estimates of b_2, b_3, and b_4 in model (7) to be

$$b_2 \triangleq 2.0,$$
$$b_3 \triangleq 2.0,$$

and

$$b_4 \triangleq -4.5.$$

Intercept and slope coefficients during calibration appeared to be about

$$b_0 \triangleq 0.5$$

and

$$b_1 \triangleq 1.0.$$

With these starting values, model (7) was fitted to the data of Table 1 using the NLIN procedure of the Statistical Analysis System (Goodnight 1979)—further details of fitting are available from the authors. The resulting, approximate least squares fit was

TABLE 1. December 1977–June 1981 stormflow volumes for 53 individual storms on Bradford WS 1 (y) and WS 3 (x), with the dummy variable (t) indicating time of harvest.

Date					
Year	Month	Day	y	x	t
			Inches	*Inches*	
1977	Dec.	17	0.007	0.006	0
		25	.005	.004	0
1978	Jan.	8	.230	.085	0
		13	.091	.036	0
		17	.003	.001	0
		19	.646	.447	0
		25	.009	.017	0
	Mar.	3	.330	.099	0
		8	.163	.070	0
		14	.007	.003	0
	Apr.	19	.063	.025	0
	May	4	3.631	2.702	0
	Jul	8	.006	.003	0
		12	.008	.003	0
		13	.416	.307	0
		16	.034	.005	0
		17	.150	.083	0
	Aug.	1	7.179	6.311	0
		5	.110	.101	0
		6	.001	.002	0
		11	.021	.021	0
		17	.005	.003	0
		18	.049	.023	0
			Harvested		
1979	Jan.	20	0.003	0.004	1
		23	.001	.001	1
		30	.007	.002	1
	Feb.	6	.031	.005	1
		24	.069	.002	1
	Apr.	5	.078	.006	1
	May	31	.002	.001	1
	Aug.	5	.006	.002	1
		23	.001	.001	1
		24	.011	.001	1
	Sept.	12	.003	.001	1
		14	.057	.002	1
		15	.641	.060	1
		24	.164	.012	1
		27	.266	.085	1
		30	.296	.048	1
	Oct.	23	.002	.001	1
	Nov.	11	.002	.003	1
	Dec.	6	.351	.261	1
1980	Jan.	12	.004	.001	1
		14	.104	.015	1
	May	25	.001	.001	1
	Jun.	19	.001	.001	1
	Jul.	25	.002	.002	1
		29	.040	.002	1

TABLE 1. Continued.

Date					
Year	Month	Day	y	x	t
			Inches	*Inches*	
1981	Feb.	11	.002	.002	1
		18	.150	.007	1
	Mar.	5	.002	.001	1
		22	.001	.001	1
		30	.004	.001	1

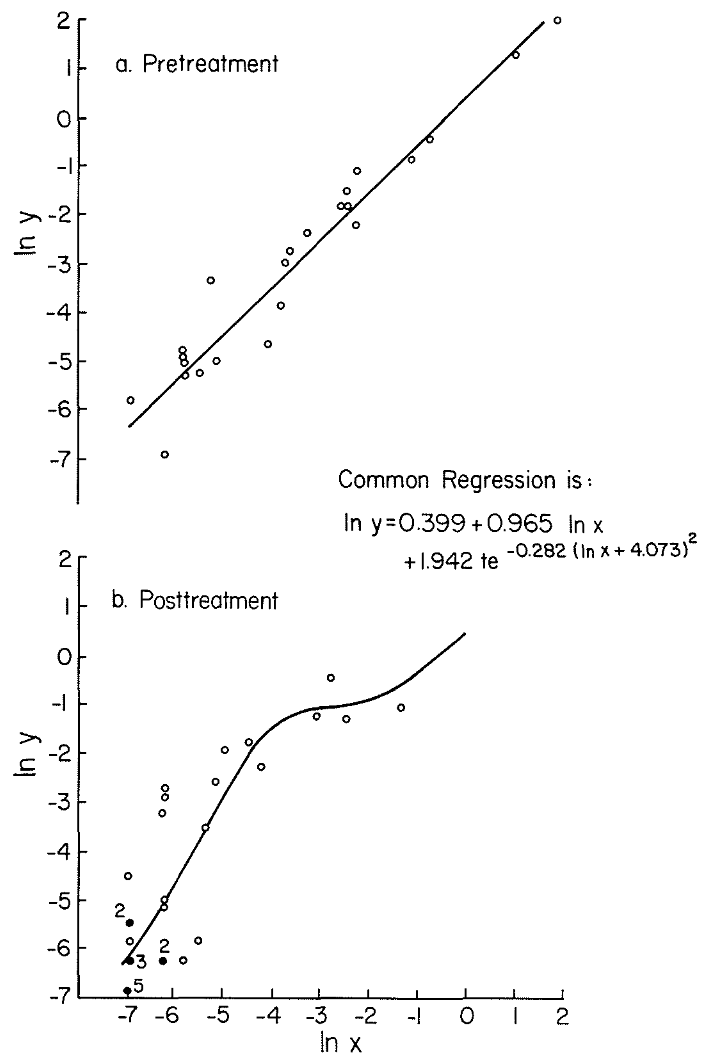

FIGURE 3. Pre- and posttreatment stormflow volumes for 53 storms on Bradford Forest Watershed 1 and Watershed 3 with fitted regression.

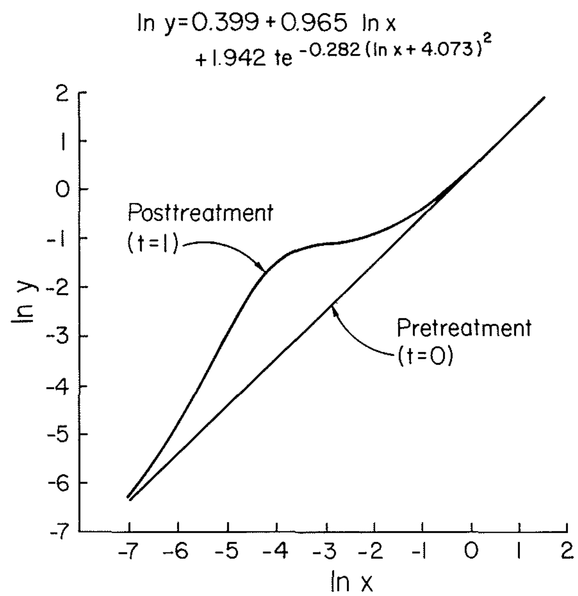

FIGURE 4. Effect of treatment on stormflow volume for Bradford Forest Watershed 1 as predicted by fitted regression.

$$\ln y_i \triangleq 0.399 + 0.965 \ln x_i + 1.942te^{-0.282(\ln x_i + 4.073)^2}. \tag{9}$$

This model is plotted against the pretreatment data in Figure 3a and against the posttreatment data in Figure 3b.

Notice in Figure 3a the aforementioned, strongly-linear relationship well fitted by (9) with $t = 0$. Compare that to the obvious curvilinear posttreatment relationship well fitted by (9) with $t = 1$. Finally, to visualize the treatment effect indicated by the model (9), both pre- and posttreatment predictors based on (9) are plotted in Figure 4.

APPROXIMATE TESTS OF SIGNIFICANCE

To test the null hypothesis of no treatment effect in any of the models (5), (6), or (7) is to test the null hypothesis

$$H_0{:}b_2 = 0. \tag{10}$$

For linear models, an exact test (Swindel 1970, Hewlett and others 1977, Gujarati 1970) of such a hypothesis is

$$F = \frac{f}{f^* - f}\,\frac{\text{SSE}^* - \text{SSE}}{\text{SSE}}, \tag{11}$$

where: f is error degrees of freedom and
SSE is error sums of squares based on the full model;
f^* is error degrees of freedom and
SSE* is error sums of squares based on the model with the hypothesis (10) incorporated.

When H_0 is true, F is distributed as Snedecor's F, with $(f^* - f)$ and f degrees of freedom. We assume the test to be approximate for nonlinear models.

To illustrate the required computations, the model (7) was fitted to the data of Table 1 in the previous section—resulting in Equation (9).

Error degrees of freedom and sums of squares for the full model are

$$f = 53 - 5 = 48,$$

and

$$SSE = 40.1080.$$

Now incorporating the hypothesis (10), that $b_2 = 0$, into the model (7) results in the reduced model

$$\ln y_i = b_0 + b_1 \ln x_i. \quad (12)$$

Fitting the reduced model (12) to all the data yielded error degrees of freedom and sums of squares

$$f^* = 53 - 2 = 51,$$

and

$$SSE^* = 54.7126.$$

Computing F by (11) yields

$$F = 5.83,$$

which, compared to tabular F with 3 and 48 degrees of freedom, is significant at the 0.005 level.

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