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Fitting Daily Precipitation Amounts Using the S_B Distribution

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ABSTRACT

The log-normal, gamma, Weibull, S_B and beta distributions were fit to daily precipitation amounts for each calendar month for a 38-year period. Data are from the high precipitation zone of the southern Appalachian Mountains. The S_B distribution, a generalization of the log-normal, consistently fit the data best. There was no striking evidence that precipitation distribution differed by month. The S_B distribution also was best when fit to all precipitation days without regard to date. The gamma distribution fit rainfall amounts accumulated for two and three consecutive wet days best.

Higher order Markov chains, up through the fifth order, described our data better than lower order chains. Thus, the probability of precipitation on any day depends on what happened on at least the five previous days. The S_B distribution of precipitation amounts on all dates preceded by dry days is different from the distribution of amounts on all dates preceded by precipitation days. The S_{BB} distribution, a simple bivariate extension of the S_B, showed low correlations between amounts of precipitation on consecutive wet days and between amounts on the first and third days of three consecutive days with precipitation. The cumulative amounts of precipitation over n days are normally distributed for large n with mean $0.53n$ cm and standard deviation $1.55n^{1/2}$ cm.

1. Introduction

In fitting statistical models to precipitation amounts, primary emphasis has been devoted to the log-normal, Pearson type III (gamma), and log-Pearson type III distributions (Das, 1955; Markovic, 1965; Borgman and Amorocho, 1970; Biondini, 1976; Bruhn *et al.*, 1980). Earlier, Slade (1936) and Kimball (1938) basically derived what is now called the S_B distribution (Johnson, 1949a) and proposed it for rainfall, runoff and flood control studies.

This paper compares the fit of the S_B and four other distributions to daily precipitation amounts from a 38-year record. The effects of different seasons or months of the year on the fit of the distributions are reported, as are effects of sequences of dry and wet days and successive wet days.

The chain-dependent Markov process was proposed by Katz (1977) for modeling the sequence of daily precipitation amounts. His procedures were further tested on this data set.

2. Methods

Data consist of daily amounts of precipitation, measured at Coweeta Hydrologic Laboratory near Otto, North Carolina from June 1936 to October 1974. The gage is at the Laboratory climatological site (35°3′34″N, 83°25′51″W) and within the high precipitation zone of the southern Appalachian Mountains. The site is near the mouth of a bowl-shaped mountain valley at an elevation of 685 m MSL. Mean annual precipitation at this gage is 1800 mm, of which ~5% is snow. Although snow data are included, for brevity all precipitation is herein referred to as rain. Measurements were made to the nearest 0.25 mm (0.01 inch). No distinction was made between zero and trace rainfall dates, and both were ignored in the fitting process. A total of 5623 rain days were found in the total set of 14 824 days.

Program MLESD (Schreuder *et al.*, 1978) was used to fit two-parameter log-normal, gamma and Weibull, and three-parameter S_B and beta distributions to the data. Because rainfall patterns were expected to differ by seasons, we fit distributions to data from each calendar month and to data from 6-month seasons. We also fit the distributions to the entire data set of all precipitation days and to sets of consecutive precipitation (wet) and non-precipitation (dry) days. Two tests were used. We ranked the fits of the distributions to each data set by the log likelihood criterion and used the Kolmogorov-Smirnov (KS) criterion to test if a particular distribution fits the data satisfactorily (Massey, 1951).

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TABLE 1. Fit to the S_B distribution of monthly and yearly sets of daily precipitation at Coweeta Hydrologic Laboratory.

Period	Probability precipitation occurrence	Sample size	Rank of distributions*	Kolmogorov-Smirnov fit criterion for S_B	S_B distribution parameters γ	δ	λ (cm)	Observed maximum (cm)	Estimated median (cm)
January	0.41	508	S_B, W, G, B, LN	0.037	1.78	0.62	12.2	11.6	0.66
February	0.40	455	W, G, S_B, B, LN	0.056	1.60	0.63	11.9	11.4	0.89
March	0.43	535	S_B, W, G, B, LN	0.047	1.89	0.65	14.6	13.7	0.76
April	0.34	408	S_B, W, G, B, LN	0.033	1.67	0.61	11.6	10.7	0.71
May	0.39	478	S_B, LN, W, G, B	0.027	2.70	0.69	25.4	19.3	0.51
June	0.43	527	S_B, LN, W, G, B	0.023	2.10	0.64	12.9	11.7	0.48
July	0.47	603	S_B, LN, W, G, B	0.017	2.11	0.69	11.5	10.3	0.51
August	0.42	529	S_B, LN, W, G, B	0.042	2.29	0.65	17.4	15.3	0.48
September	0.33	405	S_B, LN, W, G, B	0.026	2.08	0.61	16.6	14.7	0.51
October	0.27	340	S_B, LN, W, G, B	0.028	2.32	0.63	23.1	19.7	0.56
November	0.31	378	S_B, W, G, B, LN	0.036	1.57	0.56	10.5	10.0	0.61
December	0.36	457	S_B, W, G, B, LN	0.030	1.62	0.59	12.4	11.9	0.74
All data	0.38	5623	S_B, LN, W, G, B	0.031**	2.28	0.65	20.2	19.7	0.59

* Distributions: S_B: Three-parameter S_B; W: Weibull; G: Gamma; LN: Log-normal; B: Beta.
** Deviates significantly from the observed distribution at $P = 0.01$.

Likelihoods are basically statements of the probability of the observed data arising from a population characterized by the parameters estimated for a distribution. The KS criterion tests the maximum departure of the fitted distribution from the observed data. This test is very sensitive in the range of the large number of observations in this data set.

3. Results

a. Daily events

Precipitation in the southern Appalachian Mountains occurs rather uniformly throughout the year. At Coweeta, the probability of precipitation on any individual day of the year is 0.38 (Table 1) and ranges from 0.27 in October to 0.47 in July. The fall months of September–November have fewer wet days, but include some of the largest daily amounts.

The S_B distribution consistently fit the daily precipitation amounts best, or was a close second, for the variety of data sets tested. These included the sets of all wet days, days in winter, summer, each month, and also special series such as three consecutive wet days, days preceded by wet dates and days preceded by dry dates.

The S_B distribution has the density function

$$f(X) = \{\delta\lambda/[(2\pi)^{1/2}X(\lambda - X)]\} \times \exp[-0.5\{\gamma + \delta \ln[X/(\lambda - X)]\}^2], \quad (1)$$

where, in this application, X is daily precipitation amount ($0 < X < \lambda$), γ and δ are both shape and scale parameters ($-\infty < \gamma < \infty$, $\delta > 0$), and λ is the maximum precipitation parameter ($\lambda > 0$) of the distribution. Of the monthly data sets, only in February was the S_B distribution surpassed by the Weibull and gamma (Table 1). The KS test statistic did not reject the S_B distribution for any of the 12 months. Looking for seasonal trends, parameters γ and δ of the S_B appear to be fairly similar across months, except for that γ is consistently greater than 2 for summer (May–October) and less than 2 for winter (November–April). Parameters γ and δ, unlike λ, have not yet been identified with specific physical characteristics of a population so we do not know whether the seasonal contrast is meaningful. The considerable differences in estimated maximum rainfall λ between months follows from the magnitudes of the observed maximum rainfalls. May, August and October each had a very large rainstorm. There is no particular reason for large storms to only occur in the specific months where they were encountered in this data set. Clearly, the estimates of λ and γ are positively correlated. Other than the seasonal differences in γ and λ, there are no obvious indicators of differences between months in distributions of daily rainfall.

Typically, summer rains are intense, short-duration convection storms, whereas winter precipitation is usually from less intense and longer duration frontal storms. Fitted parameters of both the S_B and Weibull distributions are different for summer and winter data (Table 2). Again, the S_B distribution was judged by the likelihood criterion ($\ln L$) to have the best fit, and as before the estimates of λ and γ parameters were largest for summer days. The summer season includes all three of the months with unusually large daily values. Observed and predicted probabilities are higher for the 0–1 cm class in the summer while the winter data have higher probabilities in the middle classes. The S_B distributions which were fit to July and December data illustrate (Fig.

TABLE 2. Observed and predicted probabilities where Weibull and S_B distributions were fitted to daily precipitation amounts for winter and summer seasons.

	Probability Summer: May–October			Probability Winter: November–April		
Rainfall class (cm)	Observed	Predicted S_B	Predicted Weibull	Observed	Predicted S_B	Predicted Weibull
0–1	0.659	0.683	0.625	0.548	0.590	0.539
1–2	0.169	0.151	0.186	0.183	0.170	0.203
2–3	0.077	0.065	0.083	0.112	0.084	0.104
3–4	0.036	0.035	0.041	0.065	0.050	0.057
4–5	0.026	0.021	0.021	0.038	0.033	0.033
5–6	0.014	0.014	0.012	0.024	0.022	0.020
6–7	0.008	0.009	0.007	0.008	0.016	0.012
7–8	0.005	0.006	0.004	0.011	0.012	0.008
8–9	0.002	0.005	0.002	0.007	0.008	0.005
9–10	0.000	0.003	0.001	0.001	0.006	0.003
10–11	0.002	0.002	0.001	0.001	0.004	0.002
11–12	0.001	0.002	0.000	0.001	0.003	0.001
12–13	0.000	0.001	0.000	0.000	0.001	0.001
13–14	0.000	0.001	0.000	0.000	0.000	0.001
14–15	0.001	0.001	0.000			
15–16	0.000	0.000	0.000			
16–17	0.000	0.000	0.000			
17–18	0.000	0.000	0.000			
18–19	0.000	0.000	0.000			
19–20	0.001	0.000	0.000			
Distribution parameters:						
n	2882			2741		
S_B						
γ		2.48			1.82	
δ		0.66			0.62	
λ		21.43			13.91	
$\ln L$		−3006			−3697	
KS		0.0250			0.0420	
Weibull						
β			0.97			1.33
c			0.78			0.81
$\ln L$			−3078			−3728
KS			0.0325			0.0152

1) this seasonal contrast. To better define the shape of the curves, probabilities were calculated for smaller classes than used for the tables.

When we fit the distributions to all data, ignoring months and seasons (Table 1), the S_B fit best but the KS test showed that the fit was statistically weak. That is, calculated KS = 0.037 for the S_B distribution, whereas the KS criterion for the large sample size of 5623 is 0.022 at $P \doteq 0.01$. Because the KS test is so sensitive at this point, we feel that the S_B may be considered to fit the entire data set just as well as it fits seasonal and monthly data. It fits better than any of the other distributions tested.

The S_B distribution, in addition to fitting these data, has an easily computed median rainfall $\{=\lambda/[1 + \exp(\gamma/\delta)]\}$, where λ, γ and δ are the fitted parameters of the distribution. The simplest way to compute mean rainfall is to multiply computed probabilities by mid-points of rainfall classes. For heavily skewed frequency distributions, the median is often preferred to the mean as a measure of central tendency. The observed and predicted fit of the distribution to the entire data set is given in Table 3.

b. Sequences of days

In addition to considering the distributions for individual daily amounts, we also worked with various sequences of days. We examined whether occurrence of precipitation is a function of occurrence on up to five previous days and whether daily amounts are functions of occurrence of precipitation on the previous day. We also looked at amounts of rain accumulated for two or three consecutive wet days.

Katz (1977) used a Markov chain model for sequences of dry and wet days, finding that only a first-order model was necessary. Bruhn *et al.* (1980) found the second order appropriate only for their August

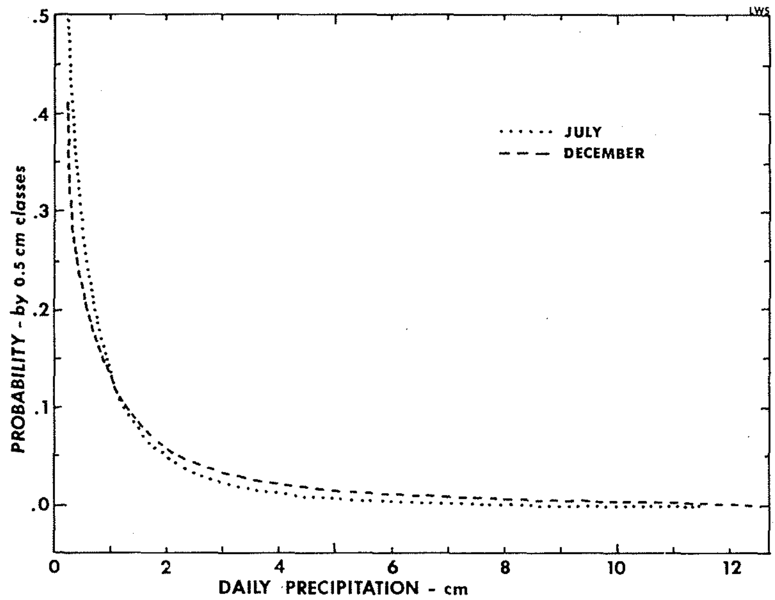

FIG. 1. S_B distributions of daily precipitation for July and December at Coweeta Hydrologic Laboratory illustrate the typically higher probabilities during the summer months for amounts $\lesssim 1.0$ cm. The estimated median precipitation for December is 0.23 cm greater than that for July.

data. Using the same testing procedures as Katz (Anderson and Goodman, 1957) on Coweeta data, we found that the probability of rain on a given day is significantly affected by the presence or absence of rain on each of at least the five previous days. For instance, probability of rain after five consecutive dry days is 0.263 while it is 0.439 after five consecutive wet days. We did not test Markov chains greater than fifth order because of the very small sample sizes for some probabilities.

TABLE 3. Observed and predicted probabilities where the S_B distribution was fitted to daily precipitation amounts for a 38-year period.

Rainfall class (cm)	Probability: Observed	Probability: Predicted
0–1	0.604	0.641
1–2	0.176	0.160
2–3	0.094	0.073
3–4	0.050	0.041
4–5	0.032	0.025
5–6	0.019	0.017
6–7	0.008	0.012
7–8	0.008	0.008
8–9	0.004	0.006
9–10	0.001	0.005
10–11	0.002	0.003
11–12	0.001	0.003
12–13	0.000	0.002
13–14	0.000	0.001
14–15	0.000	0.001
15–16	0.000	0.001
16–17	0.000	0.000

Distribution parameters: $n = 5623$; $\gamma = 2.28$; $\delta = 0.65$; $\lambda = 20.2$ cm; Median = 0.59 cm; Mean = 1.47 cm

Five statistical distributions were fit for two cases specified by occurrence or non-occurrence of precipitation on the previous day. Again, the S_B distribution was best. Table 4 shows the observed and predicted probabilities for rainfall amounts when the previous day was either dry or wet. Bruhn *et al.* (1980) show that rainfall amount is independent of rain occurrence on the previous day. However, a two-tail KS test indicates dependence by showing the two distributions in Table 4 to be significantly different at the 0.01 level. Chin and Miller (1980) found such distributions to be different only for winter data from stations in the Pacific Northwest states, while Buishand (1978) found mean daily rainfall at six stations to be dependent on the presence or absence of rain on adjacent days. For the Coweeta data, the probability of light rain (<1 cm) is higher when the previous day was dry (0.670 vs 0.612). The probabilities for classes above 1 cm are all greater if the previous day was wet. The median rainfall (1.68 cm) and mean rainfall (1.52 cm) are both greater when the previous day was wet than when the previous day was dry (1.29 and 1.22 cm).

Katz (1977) assumed that amounts of rainfall on consecutive wet days are independent. This is partly confirmed through our fitting of the S_{BB} [a generalization of the S_B (Johnson, 1949b)] to daily rainfall

TABLE 4. Observed and predicted probabilities where S_B distributions were fitted to daily precipitation amounts given that the previous day was dry or that the previous day was wet.

	Probability previous day dry		Probability previous day wet	
Rainfall class (cm)	Observed	Predicted	Observed	Predicted
0–1	0.632	0.670	0.580	0.612
1–2	0.179	0.153	0.173	0.166
2–3	0.086	0.068	0.101	0.078
3–4	0.043	0.038	0.057	0.045
4–5	0.030	0.023	0.033	0.028
5–6	0.015	0.015	0.022	0.019
6–7	0.005	0.010	0.011	0.014
7–8	0.005	0.007	0.010	0.010
8–9	0.003	0.005	0.006	0.007
9–10	0.001	0.003	0.001	0.005
10–11	0.000	0.002	0.003	0.004
11–12	0.000	0.002	0.002	0.003
12–13	0.000	0.001	0.000	0.002
13–14	0.000	0.001	0.000	0.002
14–15	0.000	0.000	0.001	0.001
15–16	0.000	0.000	0.000	0.001
16–17	0.000	0.000	0.000	0.001
Distribution parameters:				
n	2598		3025	
γ		2.18		2.21
δ		0.64		0.65
λ		15.88		20.50

amounts for pairs of consecutive wet days and for the first and third days of the set of three consecutive wet days. With the S_B distribution, the variable $\gamma + \delta \ln[X/(\lambda - X)]$ is a normal variant and thus the correlation between two distributions can be tested for significance by the t-test. Both the correlations between amounts on first and second days (0.095) and amounts on first and third days (0.022) are near zero (Table 5). Although $\hat{\rho} = 0.095$ is highly significant ($P < 0.001$) for consecutive wet days, it is so small that for most practical purposes it could be treated as zero. Despite the small correlations, note that each of the three parameters of the distributions for first and second days of a 2-day series of wet days are essentially identical. Parameters for distributions of the first and third days of a 3-day series are similar to those from the 2-day series, with the exception that the λ fit to the first days is notably less.

We also fit the same five distributions to amounts of rain accumulated for two and three consecutive wet days. For the cumulative amounts of rain on two consecutive days, the gamma distribution fit best according to the log likelihood criterion. For three consecutive days, the S_B was slightly better than the gamma. The observed probabilities and predicted fits for both sets of data for the gamma and S_B distributions are given in Table 6. The KS criterion at $P = 0.01$ for $n = 3025$ is 0.0296 and for $n = 1539$ is 0.0415.

Katz (1977) gives procedures for computing the distribution of amount of rain over n days. If

$$S_n = \sum_{i=1}^{n} X_i$$

is the cumulative amount of precipitation over n days, where X_i (precipitation on day i) has an S_B distribution, the exact distribution of S_n is too complex to be usable even with the recurrence equations given by Katz. As an alternative, using his asymptotic normal results for S_n, we have estimated mean total rainfall for n days to be $\hat{\mu} = 0.53n$ cm with estimated standard deviation $\hat{\sigma} = 1.55n^{1/2}$ cm.

4. Discussion

Records of daily precipitation amounts are difficult to analyze because the values are heavily skewed toward zero. Daily amounts from a 38-year record collected in the high-precipitation region of the southern Appalachian Mountains were fit to five distributions. The Kolmogovov-Smirnov criteria showed that the S_B distribution generally fit the data. In addition, log-likelihood criteria showed that the S_B distribution fit the data better than the log-normal, Weibull, beta or gamma distribution in nearly every case. This was true for the entire data set of all daily values without regard to date as well as the subsets of all daily values within the May–October season, within the November–April season, all daily values within most months, and all daily values given the previous day wet and given the previous day dry. The parameter values of the S_B distribution did not differ much between the various data sets, except that the γ parameter was slightly less for the November–April season and for the individual winter months. Parameter differences seemed related to the magnitudes of the observed maximum amounts.

Predicted probabilities well describe the observed precipitation phenomena. In the southern Appalachian Mountains, summer rains include a few big

TABLE 5. S_B Distribution parameters for and correlation between first and second or first and third consecutive wet days.

	First and second days of wet period		First and third days of wet period	
Parameters	First days	Second days	First days	Third days
γ	2.22	2.21	2.10	2.29
δ	0.65	0.65	0.65	0.66
λ	20.52	20.50	14.71	19.68
Sample size	3025		1539	
Correlation	0.095		0.022	
Significance level of correlation	<0.001		0.377	

TABLE 6. Observed and predicted probabilities where the gamma and S_B distributions were fitted to daily precipitation amounts accumulated for two and three consecutive days of rain.

	Probability Two consecutive days of rain			Probability Three consecutive days of rain		
Rainfall class (cm)	Observed	Predicted gamma	Predicted S_B	Observed	Predicted gamma	Predicted S_B
0–2	0.481	0.474	0.516	0.277	0.279	0.305
2–4	0.281	0.278	0.242	0.319	0.297	0.295
4–6	0.125	0.134	0.112	0.183	0.192	0.169
6–8	0.058	0.062	0.059	0.096	0.110	0.097
8–10	0.029	0.028	0.033	0.060	0.059	0.057
10–12	0.013	0.013	0.018	0.031	0.031	0.034
12–14	0.006	0.006	0.010	0.011	0.016	0.020
14–16	0.004	0.002	0.006	0.010	0.008	0.011
16–18	0.001	0.001	0.003	0.008	0.004	0.006
18–20	0.001	0.000	0.001	0.002	0.002	0.003
20–22	0.001	0.000	0.000	0.002	0.001	0.002
22–24	0.000	0.000	0.000	0.001	0.000	0.001
24–26	0.000	0.000	0.000	0.001	0.000	0.000
Distribution parameters:						
n	3025			1539		
Gamma						
α		1.22			1.67	
β		2.36			2.54	
lnL		−3370			−2221	
KS		0.0167			0.0268	
S_B						
γ			2.03			2.12
δ			0.84			1.00
λ			23.63			30.19
lnL			−3388			−2219
KS			0.0341			0.0359

storms but otherwise tend to be short thundershowers, whereas winter precipitation events are more sustained. Thus, the fitted maximum parameter of the summer distributions is large yet the probability of small daily amounts is greater than in winter. In the observed data, the typically large storm tends to cover more than one day. Similarly, predicted probabilities are greater in the larger daily amounts for wet days following wet days, whereas probabilities are higher for small amounts when wet days follow dry days. The maximum for the first day of a 3-day precipitation period was found to be less than the maximum for the first of a 2-day period. The data were not classified by storm type but appear to represent the fact that intense thundershowers may occur on two successive days, whereas longer storm periods tend to exclude the conditions which produce heavy initial rainfall. The approximation for the long-term sum, $0.53n^{1/2}$, yields 193.4 cm for the annual total which compares closely with the station mean of 182.4 cm. The gamma distribution is as good as or better than the S_B for 2- and 3-day cumulative amounts, apparently because of a better fit in the smaller rainfall classes. Either distribution matches the observed probabilities in the middle rainfall classes while the tail of the S_B extends further for the larger storms. Thus, the S_B may be preferable for all cases except when light rains extend over several days.

The log-normal is a special case of the S_B distribution in that the S_B approaches the log-normal as the maximum parameter λ approaches infinity. The ranking pattern of distributions in Table 1 places log-normal second after the S_B for the May–October months and last after all other distributions the remainder of the year, yet the difference in λ is numerically small. The sequences of ranking of the five distributions given in Table 1 are regular and show a seasonal shift. The significance of these patterns was not tested.

The S_B distribution appears to provide the best overall fit for daily precipitation amounts in the southern Appalachian Mountains. Fitting a distribution successfully to daily precipitation data facilitates several uses. Means and medians may now be calculated for these skewed data. Frequency of occurrence can be predicted for various storm sizes. For these data, for example, the probability of a storm greater than 6 cm occurring on a given day is 0.043, greater than a 10 cm storm is 0.012, and greater than a 20 cm storm is 0.000 on the basis of the fitted S_B distribution. The probability of having three consecutive days of rain is 0.104. If 3-day storms are of interest, the probability of a total amount of rain of 6 cm or greater is 0.232, more than 10 cm is 0.063, and more than 20 cm is 0.002 using the gamma distribution. The S_B yields similar values. Given the three parameters of the S_B distribution (γ, δ and λ) the frequency of occurrence of any rainfall interval can be calculated by integrating Eq. (1) over that interval.

Distributions fit to daily precipitation amounts provide criteria for simulation of a precipitation sequence such as given by Bruhn *et al.* (1980). The treatment of this data set yielded probabilities of precipitation occurrence on any day, and on a day given the previous day is dry or the previous day is wet. Using the Markov chain, probability of precipitation is defined by the status of the previous 5 days. The S_B distributions provide the probabilities of precipitation amounts on those wet days given the month, the status of the previous day, and the cumulative amounts if two or three consecutive wet days are indicated. The creation and testing for validity of a simulated record based on these constraints is beyond the scope of this paper. The purpose here is to demonstrate the availability of a useful distribution for daily precipitation amounts.

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