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ESTIMATING SOLAR RADIATION ON MOUNTAIN SLOPES

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ABSTRACT

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The amount of solar irradiation on a mountain slope is an important parameter for describing the climatology of a sloping site, but measurements of such radiation are not easily obtained. Daily totals of solar irradiation can be estimated by using published tables of potential solar irradiation if the effects of atmospheric transmissivity and cloud cover are included. Daily totals of solar irradiation ($K{\downarrow}s$) were estimated as:

$$K{\downarrow}s = K{\downarrow}(\mathrm{Sps}/\mathrm{Sp})$$

where Sps and Sp are the table values of potential solar irradiation for a slope and for a horizontal surface and $K{\downarrow}$ is the daily total of global solar radiation measured on a horizontal surface at a site near enough to have the same cloud cover as the mountain slope. A test of this method shows that valid estimates of solar radiation input were obtained for two opposite-facing slopes.

INTRODUCTION

The intensity of solar radiation has been used as a variable to represent some of the effects of mountain topography upon snowmelt, evapotranspiration, plant or animal activity, and soil type (Byram and Jemison, 1943; Baumgartner, 1960; Lee, 1963, 1964; Geiger, 1965, Sect. 41 and 44; Jackson, 1967; Famiglietti, 1969; Rouse and Wilson, 1969; Meiman et al., 1971). Measurements of, and formulas for computing, solar irradiation of slope are not new (Wiener, 1877; Milankovitch, 1930; Okanoue, 1957; Geiger, 1965, Sect. 40), but now the estimation of solar irradiation on mountain slopes has been expedited by tables for obtaining either instantaneous values (Fons et al., 1960) or daily and annual totals (Frank and Lee, 1966; Buffo et al., 1972).

Unless modified, the equations and tables will provide estimates of the intensity of solar radiation on a slope as if clouds and absorption and scattering by the atmosphere did not exist. Practical use of these estimated values of solar radiation is limited because the basic estimates do not represent the real energy conditions at the earth's surface. The estimates can be used as index values for comparing the solar radiation climates between slopes exposed to the same cloud pattern (Swift, 1960; Lee, 1963, 1964; Lee and Baumgartner, 1966; Jackson, 1967; Hendrick et al., 1971; Meiman et al., 1971). Atmospheric transmission effects, other than those resulting from clouds, can be estimated from measurements of surface weather and solar altitude (Fons et al., 1960; Garnier and Ohmura, 1968), but the variation in solar irradiation caused by clouds is not easily estimated by empirical formulas.

Solar energy is the forcing function for most of the physical and biological processes occurring at the earth's surface. A simple method of estimating solar irradiation of mountain slopes is needed as a climatological parameter for modeling energy-dependent processes such as plant growth or evapotranspiration. We have tested a method which provides for the effects of both cloud cover and atmospheric transmissivity

METHOD

Each record which is made of the intensity of solar radiation at various regional climatic stations and at some other sites is representative of the actual transmission of solar radiation through the atmosphere and the effects of cloud cover which are typical for areas near the individual station. To take advantage of this cloud information while making estimates of actual solar irradiation of two specific slopes, we multiplied the daily totals of solar radiation recorded at a climatic station in the same region by slope factors for each slope. Our technique is similar to that given by Schram and Thams (1967).

The first step was to compute a set of slope factors to convert the radiation for a horizontal surface to estimates of the radiation on a sloping surface. Frank and Lee's (1966) tables of daily totals of potential solar radiation were used to obtain slope factors for 24 days of the year. The slope factors vary over the year with solar declination. Potential or extraterrestrial solar radiation is the theoretical value of solar radiation calculated to strike a surface on the earth without the losses resulting from passage through the atmosphere.

Each factor was computed as the ratio of the daily total of potential solar radiation for the slope to the daily total of potential solar radiation for a horizontal surface. 24 slope factors were sufficient to define an annual curve; values from other days of the year were interpolated from a graph of this curve. For the purposes of this test, slope factor curves were produced for a pair of 37 % slopes, one facing north and the other facing south (Fig.1).

These particular aspects and slope were chosen so that measurements of actual solar radiation already available for two such slopes (Swift, 1972) could be used to test the estimating procedure. At these slopes, incoming total solar radiation was measured by checkerboard pyranometers (Gay and Knoerr, 1973) which were oriented parallel with the sloping land surface. The pyranometers were sampled by a digital recording system at 5-min intervals during the daylight period on 26 days which represented a range of seasons and cloud conditions. Integrated totals for each morning and afternoon were computed from the 5-min samples. By dividing the day at solar noon, the effect of afternoon cloud development could be segregated.

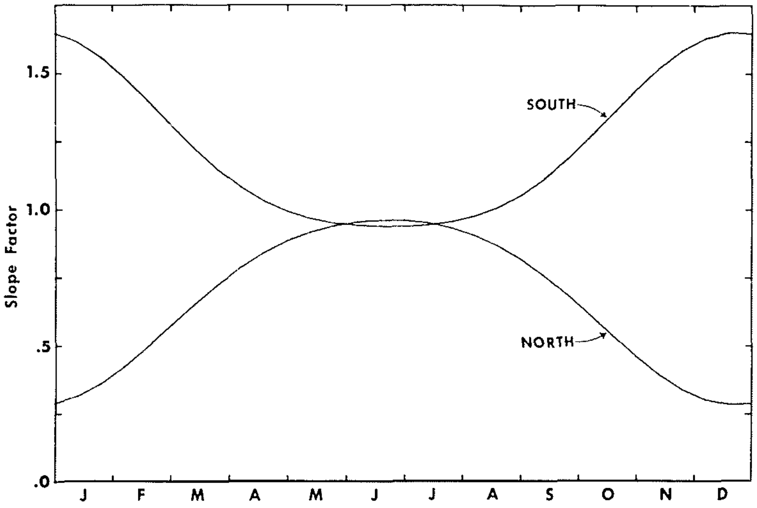

Fig.1. Calculated slope factors for south- and north-facing 37 % slopes at latitude 35°N.

All measurements were made at the Coweeta Hydrologic Laboratory in western North Carolina (35°4′N 83°26′W). The Laboratory climatic station, with an Eppley* pyranometer, was on the valley floor at 686 m (2,250 ft.) above sea level. The slope measurements were made at 1,175 m (3,855 ft.) near the ridgeline about 2.8 km (1.7 miles) away from the climatic station.

The checkerboard pyranometers were thermopile sensors designed and assembled at Duke University by Dr. L. W. Gay.** Each pyranometer was calibrated against the Eppley pyranometer at the climatic station and two Kipp pyranometers. The standard deviation from these calibration equations ranged from 0.02 to 0.05 cal. cm^{-2} min^{-1}.

RESULTS

Fig.2 is a scatter diagram of the estimated versus measured solar radiation. Each plotted point represents one-half of the daylight period, either sunrise

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* Trade names are given for the convenience of the reader and do not imply endorsement by the Forest Service or the U.S. Department of Agriculture.

** Presently at the School of Forestry, Oregon State University, Corvallis, Oregon (U.S.A.).

to solar noon or noon to sunset. The data define a line and are well-grouped except for the greater scatter in the larger values. The points in Fig. 2 represent a full range of conditions: clear, partly cloudy, and overcast skies in all seasons of the year, and both north and south slopes. Thus, the fit of the points suggests that potential irradiation, adjusted by solar radiation measured at a representative horizontal site, forms a reasonable estimate of solar irradiation for any mountain slope. The mean difference between the estimated and the measured solar radiation for the slopes was —7.7 cal. cm^{-2} per morning or afternoon period. This is a significant difference at the 95% level. Although the measurement of solar radiation on a horizontal surface can be used alone as an estimator of irradiation of the slope, the slope factor cuts both the mean absolute difference and the standard deviation of the individual differences about in half (Table I).

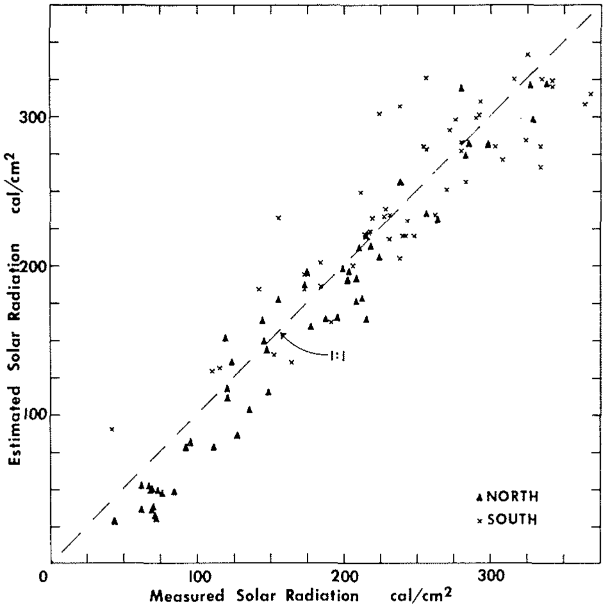

Fig. 2. Scatter diagram of estimated versus measured solar radiation on north and south slopes. Estimate equals the morning or afternoon totals of solar radiation, measured at the climatic station, times a slope factor.

Global solar radiation, measured at the climatic station, is the sum of the direct beam irradiation from the sun and the diffuse solar radiation from the sky and clouds. Because the slope factor is based upon the trigonometry of the sun's position and the angle with which the solar beam strikes the slope and because the daily total diffuse radiation is similar for all the slopes (Kondratyev and Manolova, 1960), a more accurate estimate would result from multiplying the slope factor by the direct beam component alone and then adding the diffuse component.

Because these direct and diffuse components of solar radiation were not individually measured at the climatic station, a method proposed by C. A. Federer (personal communication, 1972) was used to estimate the direct beam fraction. His method assumes that the fraction of global radiation resulting from the direct beam can be approximated by the ratio of measured to potential solar radiation for horizontal surfaces. This relationship is a linearization of the curve in fig.7 of Liu and Jordan (1960). Estimated slope irradiation was recomputed by using these approximations of the direct and diffuse components of the global solar radiation measured at the climatic station. The separate accounting for diffuse radiation and the multiplying of only the direct beam by the slope factor produced estimates of incoming solar radiation which reduced the mean difference between estimated and measured values but only slightly reduced the absolute difference and the standard deviation as compared to the estimates obtained as products of global solar radiation and the slope factor (Table I).

TABLE I

Differences between estimated and measured solar irradiation on slopes (cal. cm^{-2} per half-day)

Slope estimate based on global solar radiation measured at climatic station	Differences*		
	mean algebraic ($\bar{D}$)	mean absolute ($|\bar{D}|$)	st. dev. (S_D)
Global	5.0	45.1	55.27
(Global) (slope factor)	−7.7	23.9	29.06
(Direct) (slope factor) + diffuse	−2.2	20.9	27.98
(Global) (slope factor) + toposhade factor	0.1	18.3	24.49

*D = estimated — measured.

All the points in Fig.2 do not fit the 1:1 line. The estimated values for low-radiation days are too small for the north slope. Because about half of the low points represent midwinter days with clear skies, the recomputation for diffuse radiation did not raise those estimates. Time of day also affected the sign of the difference between measured and estimated solar radiation since morning estimates were consistently low.

The climatic station was in a valley bottom and was shaded by surrounding topography in early morning and late afternoon. The slope sites were also shaded by topography but only in the afternoon. The high ridge to the west had a greater effect on the south slope than on either the north slope or the climatic station. To test whether adjustments for topographic shading might improve the estimating method, trial "toposhade" (topographic shading) factors were assumed and added to each slope estimate. The values for three factors were chosen so that they minimized the mean differences between the estimated and the measured values for each slope's morning or afternoon set of data. 22 cal. cm^{-2} were added to each morning's estimate in order to adjust for the shading that occurred at the climatic station but not on the elevated slopes. Afternoon values were adjusted by adding 6 cal. cm^{-2} to the north estimate or subtracting 15 cal. cm^{-2} from the south estimate in order to account for the relative difference in time of shading between the climatic station and the two slope sites. There are more accurate means to determine toposhade factors (Baumgartner, 1960; Lee and Baumgartner, 1966; Ohmura, 1970), but even this simple method reduced the mean difference and the standard deviation more than did the model which separated diffuse and direct beam radiation (Table I and Fig.3). Because of large standard deviations, the shifts of the mean algebraic difference between the several models were not statistically significant. The reduction of the standard deviation from 55.27 to 29.06 through the use of the slope factor was a significant change.

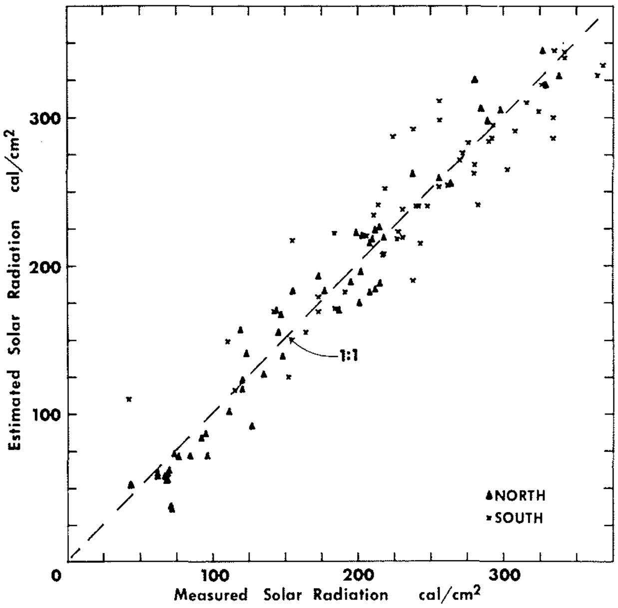

Fig. 3. Scatter diagram of estimated versus measured solar radiation on north and south slopes. Estimate equals the morning or afternoon total of solar radiation, measured at the climatic station and adjusted for topographic shading, times a slope factor.

DISCUSSION

Without making onsite measurements, the intensity of solar radiation on mountain slopes can be estimated by using slope factors derived from tables and measurements of global solar radiation on a horizontal surface if these measurements are made under cloud and atmospheric conditions similar to those at the slopes. Where day length is considerably modified by topography, the estimate is improved by using toposhade factors.

Although previously cited methods may provide a more accurate estimate of solar irradiation on slopes, the simple method described here produced estimates accurate enough for many climatological applications such as modeling the energy availability for a mountain watershed. For example, the majority of the estimated points in Fig.3 are within 10% of the measured value, which approaches the accuracy with which solar radiation routinely can be measured (Gates, 1965; World Meteorological Organization, 1971). Reasonable estimates were possible without special treatment of the diffuse component of solar radiation, even though the test period included a wide range of cloud types and atmospheric transmissivities. The effects of sky conditions were incorporated by including in the prediction equation a measurement of global solar radiation at a nearby site. Meiman et al. (1971) also show only a slight gain in prediction ability through the separate use of diffuse radiation.

Several workers have used tables of theoretical solar radiation to compute radiation indices for mountain slopes. Sky conditions were not incorporated in these indices, which were used only to make comparisons of the relative radiation load between adjacent mountain slopes. By showing that the same tables of radiation can be used to estimate solar energy under varying cloud conditions, the present report also demonstrates that a radiation index technique is valid where slopes are exposed to the same cloud pattern. The user should recognize, however, that during a season when solar radiation theoretically should differ between slopes, frequent heavy clouds will cause the actual difference in radiation received by the slopes to be less than the radiation index would indicate.

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