A COMPARISON OF SEVERAL STATISTICAL MODELS IN FOREST BIOMASS AND SURFACE AREA ESTIMATION

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ABSTRACT

The squared correlation and log likelihood techniques are discussed and used to evaluate statistical estimation models for eastern white pine biomass and surface area data. Three a priori linear models are considered: (1) an unweighted untransformed model, (2) a weighted untransformed model, and (3) a log-log transformation model. In addition, a comprehensive model is considered which includes two of the a priori models and other standard models as special cases. The likelihood approach is demonstrated to be a useful statistical tool for comparing dimensional tree component models. The $R^2$ criteria can be misleading. Specifically $R^2$ gave misleading results in selecting the unweighted untransformed model over the weighted untransformed model in five out of eight cases. Considering only the a priori models, the biomass parameters would be estimated by the log-log transformation and the weighted linear models would be used for the surface area parameters. The double square root transformation appears to be a promising alternative model.

INTRODUCTION

A knowledge of the biomass and surface area dimensions of forest stands is germane to an understanding of the major ecosystem processes which function on forest land, such as cycling of water, energy, and nutrients. With the advent of the International Biological Program, collection of dimensional tree data for a variety of forest types has been intensified and a background of information is rapidly accumulating. The acquisition of such data is laborious and involves a considerable expenditure of time and funds, even when the most simple techniques and methods available are used. Thus it appears desirable, and indeed pertinent, that the data receive adequate statistical analysis as well as adequate planning for collection.
Based on the literature, it appears that the most common procedure for estimating the biomass of coniferous and hardwood forests employs sample trees which frequently are separated into component parts; i.e., branches, foliage, stem, and sometimes roots and bark (Satoo, et al., 1956; Ovington, 1957; Baskerville, 1965; Blackmon and Ralston, 1968). Relationships are obtained between tree diameter, or height, or both with a tree component dry weight; frequently the equation used to describe the relationship is of the form $y = a(x)^b$, where $y$ is the tree component weight, $a$ and $b$ are constants, and $x$ is tree diameter or some other independent variable (Kittredge, 1944; Attiwill, 1966; Whittaker and Woodwell, 1968). Although not specifically stated, it often appears that a given equation form is assumed to give the best fit for the data and other possible statistical models are not evaluated.

The objectives of this paper are (1) to describe, discuss, and demonstrate some available techniques for obtaining and evaluating statistical estimation models, and (2) to apply these techniques and determine the most appropriate model for biomass and surface area estimation for a specific set of tree data. Known (a priori) models to be compared will include:

$$Y = a + b(X) + \varepsilon \quad \text{where } \varepsilon \sim N(0, \sigma^2) \quad (1)$$
$$Y = a + b(X) + \varepsilon \quad \text{where } \varepsilon \sim N(0, \sigma^2X^k) \quad (2)$$
$$\ln Y = \ln a + b \ln(X) + \ln \varepsilon \quad \text{where } \ln \varepsilon \sim N(0, \sigma^2) \quad (3)$$

where $Y$ is tree component dry weight or surface area and $X$ is tree basal area at breast height or a combination of basal area and total tree height.

**STATISTICAL EVALUATION METHODS**

Prior to development of the models mentioned above, it is appropriate to define and discuss the relevant existing statistical approaches which will be used in all comparisons between models. Because transformations are made in the variable of interest, a simple comparison of standard errors of estimates is meaningless. The usual measure of goodness of fit is the squared correlation coefficient ($R^2$). This measure is a valid one if the conditions under which it is meant to apply are met. Random sampling from a multivariate normal population is a primary requirement. However, since the values are often selected in some way other than simple random sample, this assumption often is violated. In other cases, it is known that the underlying population is not even approximately normal. For instance, our data come from a forest stand in which the distribution of tree diameters ($D$) is approximately normal. But since we sometimes work with $\log D$ or $D^2$, we know immediately that these variables are not normally distributed; therefore, the assumption is known to be violated and the concept of $R^2$ can be meaningless. Moreover, we know that the inclusion of other independent variables never decreases $R^2$ so that a
comparison of values is only useful when the number of independent variables in each model is the same. Although the $R^2$ concept is not always meaningful in our sampling situation, it is easily calculated and will be compared with other statistical approaches.

Furnival (1961) and Cox (1961) present an approach which evaluates models by comparing the product of the likelihoods of the values of the dependent variable under the different models. Cox (1961, 1962) in addition discusses the construction of significance tests in specific situations. The likelihood indicates the probability that the set of data we are working with arose from the model. The more closely the model under consideration resembles the true model, the larger the likelihood. The likelihoods provide a rational measure of belief in the models and can be called the experimental support for the models (Edwards, 1969). We often work with the natural log of the likelihoods because it simplifies calculations and does not affect the conclusions.

The likelihood approach requires the specification of a finite set of models and (in the cases treated) assumes each model has normally and independently distributed errors with zero means and constant variance. A major advantage of this approach is that the assumptions of normality, independence of residuals, and constant variance are considered and are reflected in the magnitude of the computed likelihoods. A disadvantage is that the number of coefficients to be determined affects the size of the likelihood. The likelihood approach has wide applicability in that all models can be evaluated. Also, the more common linear and nonlinear models with normally distributed errors can easily be fitted with standard least square computer programs. An additional advantage is that models must be specified prior to data analysis, thus ensuring that some forethought is given toward defining meaningful models.

Box and Cox (1964) and Cox (1968) suggested an additional approach which may be considered as a generalization of the likelihood approach. Their first step is to specify a comprehensive model which includes many models of interest as special cases. For example, an attractive possibility is the simple power transformation

\[
\frac{Y^{\lambda_2} - 1}{\lambda_2} = a + b \frac{X^{\lambda_1} - 1}{\lambda_1} + \varepsilon \quad \text{where } \varepsilon \sim N(0, \sigma^2) \tag{4}
\]

and $Y > 0, X > 0$, $-\infty < \lambda_1, \lambda_2 < \infty$

where $\frac{Y^{\lambda_2} - 1}{\lambda_2} = \ln Y$ and $\frac{X^{\lambda_1} - 1}{\lambda_1} = \ln X$

by definition for $\lambda_2 = 0$ and $\lambda_1 = 0$ respectively.
We will use this comprehensive model in our analysis because it contains 
models (1) and (3) as special cases for \((\lambda_1, \lambda_2) = (1, 1)\) and 
\((\lambda_1, \lambda_2) = (0, 0)\), respectively.

The second step of the generalized likelihood approach is the 
estimation of all parameters \((\lambda_1, \lambda_2, \alpha, \beta)\) by least 
squares analysis and the construction of a confidence region for parameter 
of particular interest. The pre-specified model most similar to the fitted 
model is then usually selected as the best model. This approach not only 
provides a look at the models of interest, but also gives the best fitting 
model within the comprehensive family of models considered. The approach 
may lead to a failure to formulate meaningful hypothesis by some researches 
but this is not a deficiency of the approach but of the users of the 
approach. Sometimes it may be difficult or impracticable to find a 
comprehensive model, for example model 4, sufficiently broad to include all 
special case models. The computations needed in the approach are not simp 
when nonlinear least squares programs are used to arrive at point estimate 
of the parameters. Because of this difficulty and because specifically we 
are more interested in the two-dimensional likelihood surface as a function 
of \(\lambda_1\) and \(\lambda_2\) than in the four-dimensional likelihood surface as a function 
\((\lambda_1, \lambda_2, \alpha, \beta)\), we will determine the likelihoods of our set of data for 
given combinations of \(\lambda_1\) and \(\lambda_2\) by means of linear least squared programs. 
This means that we will obtain the approximately best model within the 
comprehensive model (4).

All three approaches will be applied to a set of actual data to 
which we want to fit the best model. At this point, we are interested in 
the best estimation equation for the population from which the data were 
selected, and hence we will not consider prediction equations which require 
additional considerations, such as limiting conditions.

### DESCRIPTION OF BASIC DATA

The biomass and surface area data used in the analysis section of 
this paper were collected from a young white pine (Pinus strobus L.) stand 
at the Coweeta Hydrologic Laboratory in western North Carolina, U. S. A. 
The pines were planted on a 16.1 hectare south-facing watershed in 1957 
following cutting of the mixed hardwood forest indigenous to the area. 
The watershed has an average slope of 48 percent and elevations range from 
706 meters to 988 meters; rainfall averages 1,727 millimeters annually.

Twenty sample trees were selected by stratified random sampling 
from the population in February 1968, and six additional sample trees were 
collected in February 1970. The year prior to the first tree collection, 
the stand contained a mean basal area of 7.3 m\(^2\) per hectare, with a mean 
stocking of 1,780 trees per hectare and a mean tree diameter of 7.1 
centimeters. Two growing seasons later, the basal area was 12.6 m\(^2\) per 
hectare, stocking was 1,780 trees per hectare, with a mean tree diameter 
of 9.7 centimeters.
Sample trees from each collection were separated into component parts consisting of foliage, branches, and stems. In the cases of foliage and branch biomass, the sample consisted of the entire tree; subsampling was used for stem biomass. Subsampling methods were also used to determine surface area of each tree component. A detailed discussion of techniques and treatment of sample tree data for the first tree collection are given in a separate paper. Information derived from the study was applied to the second tree collection. The mean and range of tree dimensions for the combined 26 sample trees are given in Table 1.

**ANALYSIS**

The models considered in the analysis are the following:

A. \[
\frac{Y_1}{\lambda_1} = \alpha + \beta \frac{X_1}{\lambda_2} + \varepsilon \quad \varepsilon \sim N(0, \sigma^2 I)
\]

for \(\lambda_1 = -2, -1, 0, 1/2, 1, 2\) and \(\lambda_2 = -2, -1, 0, 1/2, 1, 2\).

This includes the a priori models (1) and (3) for \((\lambda_1, \lambda_2) = (1, 1)\) and \((\lambda_1, \lambda_2) = (0, 0)\), respectively. The latter is the common log-log transformation. Also included are the less common double square root \(\sqrt{1/2, 1/2}, 1, 2, 2, 2\) transformations, along with others.

The formula for computing the log of the product of the likelihoods for this comprehensive model is:

\[
\ln L = -\frac{n}{2} \ln \sigma^2 - \frac{n}{2} \ln \sigma^2 - \frac{n}{2} \ln Y_i + \sum_{i=1}^{n} \ln Y_i
\]

where \(\hat{\sigma}^2\) is the mean square residual and \(n\) is sample size.

In addition, we consider a priori model (2):

B. \[
Y = \alpha + \beta X + \varepsilon \quad \varepsilon \sim N(0, \sigma^2 X^k)
\]

where \(k\) is assumed known.

The formula for the log of the product of the likelihoods for this model is:

\[
\ln L = -\frac{n}{2} \ln \sigma^2 - \frac{n}{2} \ln \sigma^2 - \frac{n}{2} \ln \sigma^2 - \frac{k}{2} \sum_{i=1}^{n} \ln X_i
\]

---

Sample trees from each collection were separated into component
to consisting of foliage, branches, and stems. In the cases of foliage
branch biomass, the sample consisted of the entire tree; subsampling
used for stem biomass. Subsampling methods were also used to determine
face area of each tree component. A detailed discussion of techniques
treatment of sample tree data for the first tree collection are given
in Table 1. Information derived from the study was applied to
second tree collection. The mean and range of tree dimensions for the
bined 26 sample trees are given in Table 1.

ANALYSIS

The models considered in the analysis are the following:

\[ \frac{\lambda_1 - 1}{\lambda_1} = \alpha + \beta \frac{\lambda_2 - 1}{\lambda_2} + \epsilon \]

\( \epsilon \sim N(0, \sigma^2 I) \)

for \( \lambda_1 = -2, -1, 0, 1/2, 1, 2 \) and \( \lambda_2 = -2, -1, 0, 1/2, 1, 2 \).

This includes the a priori models (1) and (3) for \((\lambda_1, \lambda_2) = (1, 1)\)
\((\lambda_1, \lambda_2) = (0, 0)\), respectively. The latter is the common log-log
transformation. Also included are the less common double square root
\((\lambda_1, \lambda_2) = (1/2, 1/2)\), double square \([(\lambda_1, \lambda_2) = (2, 2)]\), and semi-log
\((\lambda_1, \lambda_2) = (0, 1)\) and \((1, 0)\) transformations, along with others.

The formula for computing the log of the product of the likelihoods
this comprehensive model is:

\[ \ln L = -\frac{n}{2} - \frac{n}{2} \ln 2\pi - \frac{n}{2} \ln \sigma^2 + (\lambda_1 - 1) \sum_{i=1}^{n} \ln Y_i \]

where \( \sigma^2 \) is the mean square residual and \( n \) is sample size.

In addition, we consider a priori model (2):

\[ Y = \alpha + \beta X + \epsilon \quad \epsilon \sim N(0, \sigma^2 X^k) \] where \( k \) is assumed known.

The formula for the log of the product of the likelihoods for this
model is:

\[ \ln L = -\frac{n}{2} - \frac{n}{2} \ln 2\pi - \frac{n}{2} \ln \sigma^2 - \frac{k}{2} \sum_{i=1}^{n} \ln X_i \]

---

1 Swank, W. T. and H. T. Schreuder, 1969. Surface area and biomass
imates for a young Pinus strobus L. plantation. Coweeta Hydrologic
oratory, Franklin, North Carolina, unpublished data and report. Presented
International Botanical Congress, Inc., University of Washington, Seattle,
ington, August 24—September 2, 1969.
In all cases, $Y$ represents any of the six tree components previously mentioned, and $X$ is either basal area (BA), diameter breast high squared times total height ($D^2H$), or diameter breast high times total height (DH), depending on the components considered. The $k$-values were determined by dividing the first twenty sample trees selected into equally-spaced diameter breast high classes, calculating the variances of the tree components within each diameter class, and fitting the relation $V = aD^b$ by least squares where $V$ is the variance and $D$ is diameter breast high (see Table 2). Since we primarily used BA as an independent variable, $k$ was determined to be 1.5 (approximately $b/2$) for all relations, except for stem area where $k = .8$ was used. Since the $k$-values were estimated from part of the data, the weighted linear model has an initial edge over the other models. Certainly the weighted should always be better than the unweighted linear models in all six of our cases. On the other hand, previous information (Young, et al., 1964; Attiwill, 1966) and experience indicates that it is quite reasonable to expect linear relations with heterogeneity of variance for biomass data. The weighting factor ($k$) might be improved by considering both the intercept and the slope in the weighting process.

Previous analysis on the basis of maximum squared correlation showed that BA was the best covariate for foliage and branch area and weight, while stem area was best estimated by DH and stem weight by $D^2H$. The fact that stem area and weight are best estimated from DH and $D^2H$ followed immediately from the way they were calculated. For comparison purposes, BA and DH were also used as independent variables for stem weight.

RESULTS AND DISCUSSION

The main results of the analysis are presented in Tables 3a and 3b for $R^2$ and log likelihood, respectively, for all models considered. In the following discussion, the log likelihood criteria will be used as the appropriate evaluation method.

Several important observations can be made from these results. First, considering only a priori models, the $R^2$ criteria selects a different model in three out of six cases. Considering all models, the $R^2$ criteria selects a different model in three out of eight cases; in one case it would have been quite misleading. The inadequacy of the criteria is demonstrated by the fact that the unweighted linear model is selected over the weighted linear model in five out of eight cases. Using the likelihood criteria, the weighted linear model was, as expected, always better than the unweighted linear model.

To gain a further understanding of the behavior of the two evaluation methods, we plotted the $R^2$ and likelihood response surfaces as a function of $\lambda_1$ and $\lambda_2$ for stem area and foliage weight (Figures 1 and 2) and for the other variables (not shown here).
In looking at the $R^2$ response surface (Figure 1), we see little change in the value if we stay on the line $\lambda_1 = \lambda_2$, but there is a sharp increase in both the $\lambda_1$ and $\lambda_2$ directions. This was true for all variables. This surface indicates that the degree of linearity between variables is the determining factor in the size of $R^2$. Although the $R^2$ surface is generally smooth, the surfaces shown indicate that more than one ridge top may occur and it is this feature of the surface that led to a misleading conclusion in the case of foliage weight.

The log likelihood response surfaces all were smooth with one ill-defined ridge top (Figure 2). The likelihood changes rapidly in the $\lambda_1$ direction but more slowly in the direction of $\lambda_2$. These surfaces show the relative importance of the violation of the previously stated assumptions underlying the likelihood model and the relative unimportance of degree of linearity. This interpretation is, however, much more true along the slopes than along the ridge tops of the surfaces.

The likelihoods in Table 3b can be utilized to select the most appropriate model for each of the six variables. On the basis of the statistical results, considering only a priori models, the biomass parameters would be estimated by the log-log transformation and the weighted linear models would be used for the surface area parameters. Considering all models, the double square root transformation appeared a promising alternative in several cases. However, the limited range of data may lead to reduced discrimination ability between transformations. This is due to the high frequency of equally or almost equally good results in both the likelihood and $R^2$ cases. In actual practice, we would use the weighted linear (untransformed) model for all variables since common sense, simplicity, and ease of interpretation would favor this model over the log-log model.

Heretofore, biomass and surface area data collected from forest stands have generally received insufficient statistical analysis. Usually transformation like the log-log relation is applied to dimensional tree variables, with little thought given to alternative models. The results of this study show that the indiscriminate use of the log-log relations is not warranted. The utility of the likelihood approach and the danger of using the $R^2$ approach have been shown when alternative models are compared. This conclusion is apparent, even though our data are limited in range and quantity. Other investigators with more information available find even more utility in the likelihood comparisons and larger discrepancies between the $R^2$ and likelihood model comparisons.
LITERATURE CITED


Table 1. The mean and range of tree dimensions for 26 eastern white pine trees sampled from a young forest stand.

<table>
<thead>
<tr>
<th>Tree Dimension</th>
<th>Mean</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basal Area, Breast Height (cm$^2$)</td>
<td>94.0</td>
<td>5.0-263.0</td>
</tr>
<tr>
<td>Total Height (m)</td>
<td>7.0</td>
<td>3.4-12.7</td>
</tr>
<tr>
<td>Foliage Weight (kg)</td>
<td>3.4</td>
<td>0.1-8.7</td>
</tr>
<tr>
<td>Branch Weight (kg)</td>
<td>10.9</td>
<td>0.3-40.2</td>
</tr>
<tr>
<td>Stem Weight (kg)</td>
<td>11.6</td>
<td>0.4-42.2</td>
</tr>
<tr>
<td>Foliage Area (m$^2$)</td>
<td>67.0</td>
<td>3.5-178.8</td>
</tr>
<tr>
<td>Branch Area (m$^2$)</td>
<td>10.3</td>
<td>1.1-33.8</td>
</tr>
<tr>
<td>Stem Area (m$^2$)</td>
<td>1.4</td>
<td>0.2-4.0</td>
</tr>
<tr>
<td>DH (dm$^2$)</td>
<td>78.6</td>
<td>9.0-216.8</td>
</tr>
<tr>
<td>$D^bH$ (m$^3$)</td>
<td>10.4</td>
<td>0.2-36.9</td>
</tr>
</tbody>
</table>

Table 2. Statistics for the relationship between the variance ($V$) of tree components and dbh classes ($D$): $V = aD^b$.

<table>
<thead>
<tr>
<th>Tree Components</th>
<th>a</th>
<th>b</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foliage Area</td>
<td>136.87</td>
<td>2.89</td>
<td>.82</td>
</tr>
<tr>
<td>Foliage Weight</td>
<td>208.75</td>
<td>3.22</td>
<td>.83</td>
</tr>
<tr>
<td>Branch Area</td>
<td>29.70</td>
<td>2.83</td>
<td>.99</td>
</tr>
<tr>
<td>Branch Weight</td>
<td>1036.</td>
<td>3.50</td>
<td>.99</td>
</tr>
<tr>
<td>Stem Area</td>
<td>0.80</td>
<td>1.64</td>
<td>.81</td>
</tr>
<tr>
<td>Stem Weight</td>
<td>0.60</td>
<td>3.66</td>
<td>.92</td>
</tr>
</tbody>
</table>
Table 3 (a). Squared correlation coefficients for relationships of tree variables under a variety of models.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Criteria</th>
<th>Best</th>
<th>Best</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-2,-2 -1,-1 0,0 1/2,1/2 1,1 1,1 weighted</td>
<td></td>
<td></td>
</tr>
<tr>
<td>X1-BA</td>
<td>.966 .962 .965 .960 .940</td>
<td>.953</td>
<td>(0,0)</td>
</tr>
<tr>
<td>Y2-BA</td>
<td>.947 .960 .937 .947 .959</td>
<td>.87</td>
<td>(0,0)</td>
</tr>
<tr>
<td>Y3-BA</td>
<td>.940 .955 .971 .969 .954</td>
<td>.960</td>
<td>(0,0), (1/2,1/2)</td>
</tr>
<tr>
<td>Y4-BA</td>
<td>.77 .99 .976 .957 .967</td>
<td>.960</td>
<td>(0,0), (1/2,1/2)</td>
</tr>
<tr>
<td>Y5-DH</td>
<td>.992 .982 .986 .987 .988</td>
<td>.985</td>
<td>(1,1), (1/2,1/2)</td>
</tr>
<tr>
<td>Y6-BA</td>
<td>.984 .954 .979 .969 .942</td>
<td>.73</td>
<td>(0,0)</td>
</tr>
<tr>
<td>Y6-D^2H</td>
<td>.985 .981 .987 .983 .967</td>
<td>.956</td>
<td>(0,0)</td>
</tr>
<tr>
<td>Y5-DH</td>
<td>.80 .88 .983 .984 .978</td>
<td>.958</td>
<td>(0,0)</td>
</tr>
</tbody>
</table>

Table 3 (b). Likelihoods (logs) for relationships of tree variables under a variety of models.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Criteria</th>
<th>Best a priori</th>
<th>Best overall model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-2,-2 -1,-1 0,0 1/2,1/2 1,1 1,1 weighted</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y1-BA</td>
<td>-267 -221 -196 -198 -204</td>
<td>-197 -227</td>
<td>(0,0), (1,1)^w, (1/2,1/2)</td>
</tr>
<tr>
<td>Y2-BA</td>
<td>-311 -254 -213 -222 -242</td>
<td>-225 -285</td>
<td>(0,0), (1,1)^w, (1/2,1/2)</td>
</tr>
<tr>
<td>Y3-BA</td>
<td>-103 -118 -100 -91 -91</td>
<td>-122</td>
<td>(1,1)^w, (1/2,1/2)</td>
</tr>
<tr>
<td>Y4-BA</td>
<td>-105 -71 -58 -40 -55</td>
<td>-37 -69</td>
<td>(1,1)^w, (1,1)^w, (1/2,1/2)</td>
</tr>
<tr>
<td>Y5-BA</td>
<td>-124 -99 -82 -78 -82</td>
<td>-78 -104</td>
<td>(1,1)^w, (1/2,1/2)</td>
</tr>
<tr>
<td>Y6-BA</td>
<td>-128 -76 -41 -51 -69</td>
<td>-48 -112</td>
<td>(0,0)</td>
</tr>
<tr>
<td>Y6-D^2H</td>
<td>-125 -74 -35 -33 -48</td>
<td>-39 -69</td>
<td>(0,0)</td>
</tr>
<tr>
<td>Y5-DH</td>
<td>-157 -89 -58 -59 -62</td>
<td>-48 -87</td>
<td>(0,0)</td>
</tr>
</tbody>
</table>

Notation

Y1 Foliage Weight, Y2 Branch Weight, Y3 Foliage Area, Y4 Branch Area, Y5 Stem Area, Y6 Stem Weight, BA Breast Height Basal Area, DH Diameter Breast Height X Tree Height, D^2H Diameter Breast Height Squared X Tree Height
Figure 1. Squared correlation response surfaces for foliage weight vs. BA and stem area vs. \(D^H\) as functions of \(\lambda_1\) and \(\lambda_2\).
Figure 2. Log likelihood response surfaces for foliage weight vs. BA and stem area vs. $D^H$ as functions of $\lambda_1$ and $\lambda_2$. 

Foliage weight vs. BA

Stem area vs. $D^H$