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INSTRUMENTAL AND SOIL MOISTURE VARIANCE USING THE NEUTRON-SCATTERING METHOD

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The neutron-scattering method for measuring soil moisture is becoming well established in field studies of moisture storage, movement, and volume changes.[2] Many experiments are being designed around the unusual precision afforded by this method in the measurement of moisture content and moisture change with time in field and forest soils. The sampling errors involved in estimates of moisture, as well as the optimum timing interval for making individual observations, have, however, been the subject of controversy among users. The experimenter must decide at the outset whether his study objectives call for maximum information about moisture at a single point in soil, or for maximum information about the average of many points in a volume of soil. Setting of timing intervals and error limits hinge on this decision, since a sampling procedure is indicated in the latter case but not in the former.

Field sampling errors, except for human mistakes in reading scaler counts and timing intervals, may be classified thus:

(*a*) Instrument error (includes error from irregular counting of returning neutrons as well as error due to electronic "noise" in the equipment).

(*b*) Timing error (random errors in measurement of the time interval during which counts are accumulated for a single observation).

(*c*) Location error (reflecting the difference in soil moisture content and soil physical properties from place to place).

As will be shown, error associated with the coefficient of calibration used to convert counts to moisture content is included in location error.

A fourth component is failure to reposition the neutron probe at the exact depth on successive measurements. If, however, proper care is used in field procedures the only effect of such error will be to add slightly to location error.

The following mathematical analysis of sampling is intended to provide the user with a systematic model for evaluating the several sources of error. Consideration of the theoretical aspects of instrument and timing errors will serve to improve experimental design of soil moisture studies in hydrology, forestry, and agriculture.

VARIANCE OF MOISTURE ESTIMATES

Assume that an estimate is to be made of the average moisture content of a soil stratum of definable size. A large number of possible sampling points may be randomly selected from the population representing the mean depth of the soil stratum.

From observations in counts obtained at n sampling points, it is generally desired to calculate a quantity c, which is the average number of counts per minute recorded during sampling. The quantity c is used as the estimate of the true population parameter C, where C is the value for mean counts per minute over the entire sampling area, that is the average counts per minute that would be obtained if complete information were available concerning the amount of moisture present in the soil stratum under study.

For any given time and population, C is, of course, a constant. A different value of c, however, will generally be obtained for every independent sample that could be taken in the stratum. The utility of the quantity c as an estimate of C therefore depends to a large degree on the uniformity (or lack of it) in values of c obtained from independent samples. As a measure of the degree of uniformity present in this estimator, it is common statistical practice to use the variance of c [denoted as Var (c)] which is defined as the expected value of $(c - C)^2$.

In designing a sampling plan, it is generally desired to minimize Var (c) for a given sampling investment. Ability to do this will necessarily

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[2] For description of the equipment and general method of operation, see Van Bavel (5).

depend on knowledge of the relationships between Var (c) and

(I) Instrument variance, which arises from the fact that the probe itself is essentially a sampling device; that is, the same instrument would not generally produce the same number of counts in two successive readings in a fixed moisture field.

(II) Timing variance, which reflects the inability of the instrument user to exactly determine the length of time during which counts were recorded by the probe. Specifically, the timer used by the operator will produce a value t which is an estimate of T—the true timing interval of sampling. The expected difference between t and T will depend on the quality of the timer and the care exercised by the operator in its use.

(III) Locational variance, which indicates the expected difference between the true counts per minute (independent of instrument and timing error) at a given sampling location and the population mean (C) for the entire area.

In order to specify the relationships between Var (c) and the above sources of variation, it is important to note that an observation on the number of counts per minute is not obtained directly from the instrument, but is derived from two more basic components: the total number of counts k recorded by the instrument, and the estimated time interval t as recorded by the timer. Specifically, counts per minute is obtained as

$$c = \frac{k}{t} \qquad (1)$$

Thus, since c is obtained as the ratio of k and t, it is possible to express Var (c) in terms of the variances and covariances of k and t as follows (3)

$$\text{Var}(c) \doteq \frac{K^2}{T^2}\left(\frac{\text{Var}(k)}{K^2} + \frac{\text{Var}(t)}{T^2}\right) \qquad (2)$$

where covariance of k and t are regarded as negligible and

$K = CT$ = population mean number of counts recorded during interval T.

Var (k) = variance of observed total count recorded during sampling period of true duration T.

Var (t) = variance of observed timing intervals in sampling period of true duration T.

In this expression, Var (t) is simply the variance of observed counting times about the true counting interval T. The term Var (k), however, is a composite variance which combines both location and instrument error. In order to specify the effect of these components, it becomes necessary to standardize the time unit in which variance will be measured. Therefore we define Var (L) as the locational variance in counts with $T = 1$ minute and Var (I) as the instrument variance in counts with $T = 1$ minute, and note that Var (k) in this case is given by

$$\text{Var}(k) = \text{Var}(L) + \text{Var}(I) \qquad (3)$$

No location-instrument covariance term is included, since, for all practical purposes, location errors and instrument errors at a single sampling point can be assumed to be independent.

Now, however, if a timing interval T minutes (not necessarily 1 minute) is used in practice, the variance of the total counts recorded during this interval is (1)

$$\text{Var}(k) = T^2\,\text{Var}(L) + T\,\text{Var}(I) \qquad (4)$$

It is important to note that expression (4) depends upon the fact that instrument error in one time interval is completely independent of the instrument error in any other time interval. Thus, the sampling of counts during successive time intervals at a single observation point consists of drawing independent samples with respect to instrument errors.

Finally, the substitution of expression (4) into (2) produces

$$\text{Var}(c) \doteq \frac{K^2}{T^2}\left(\frac{T^2\,\text{Var}(L) + T\,\text{Var}(I)}{K^2} + \frac{\text{Var}(t)}{T^2}\right) \qquad (5)$$

or, since $C = K/T$,

$$\text{Var}(c) \doteq \text{Var}(L) + \frac{\text{Var}(I)}{T} + \frac{C^2\,\text{Var}(t)}{T^2} \qquad (6)$$

Equation (6) shows the contributions of the three error components to variance of the estimated mean counts per minute. In general, however, moisture estimates will be presented in terms of moisture volume rather than counts per minute. Conversion of counts per minute to moisture volume (m) is commonly accomplished by the use of linear calibration curves of the general form

$$m = bc \tag{7}$$

where b is the slope of the calibration curve. It is assumed that b is a constant in this treatment, and that deviation of b from true B constitutes a bias rather than error in equation (7). Therefore the variance of estimates of moisture volume (m) is easily derived from equations (6) and (7) as

$$\mathrm{Var}(m) \doteq b^2\left(\mathrm{Var}(L) + \frac{\mathrm{Var}(I)}{T} + \frac{C^2\,\mathrm{Var}(t)}{T^2}\right) \tag{8}$$

The variety of methods used to arrive at the calibration coefficient b has led to some controversy over the part it plays in estimating variance of moisture content. If b were customarily derived from simultaneous sampling of m and c in the soil stratum to be routinely sampled for c alone, the error could be treated (1) as a double sampling problem requiring information about both Var (c) and Var (b). But this procedure does not correspond to present practice in field sampling, partly because the cost of double sampling each area and soil stratum is prohibitive. The cause of variance in b needs to be clarified, and some recent work is a step in this direction (2, 6). In the meantime, the influence of Var (b) on Var (m) is implicit in equation (8), where it is included in the Var (L).

VARIANCE OF ESTIMATES OF MOISTURE CHANGE WITH TIME

In most experiments in hydrology, forestry, and agriculture, moisture change with time is the variable of interest, particularly in research concerning moisture storage, evapotranspiration, drainage, and movement of water within profiles.

The variance of moisture changes with time can be easily obtained through further development of formula (6). Let

c_1 = observed counts per minute, first measurement.
T_1 = true timing interval, first measurement.
C_1 = population mean counts per minute, first measurement.
Var (I_1) = random per minute counting variance, first measurement.
c_2 = observed counts per minute, second measurement.
T_2 = true timing interval, second measurement.
C_2 = population mean counts per minute, second measurement.
Var (I_2) = random per minute counting variance, second measurement.

The variance of the difference in counts is then given by

$$\begin{aligned}\mathrm{Var}(c_1 - c_2) &\doteq \mathrm{Var}\,c_1 + \mathrm{Var}\,c_2 - 2\,\mathrm{Cov}\,c_1c_2 \\ &\doteq \mathrm{Var}(L_1) + \frac{\mathrm{Var}(I_1)}{T_1} + \frac{C_1^2\,\mathrm{Var}(t)}{T_1^2} \\ &\quad + \mathrm{Var}(L_2) + \frac{\mathrm{Var}(I_2)}{T_2} + \frac{C_2^2\,\mathrm{Var}(t)}{T_2^2} \\ &\quad - 2\,\mathrm{Cov}\,c_1c_2\end{aligned} \tag{9}$$

This assumes that Var (L) and Var (I) may be different for the two measurements, but that the Var (t) is the same for both determinations. It seems unrealistic to assume any covariance between either the first and second timing errors or instrument errors. Furthermore, since location variation is independent of both instrument and timing errors, and since timing errors are also independent, the only component contributing to the covariance term is due to location. Hence,

$$\mathrm{Cov}\,c_1c_2 = \mathrm{Cov}\,L_1L_2$$

where Cov L_1L_2 is the covariance between counts at the same location at different times. In practice, the size of the covariance term is dependent upon the true covariance existing in the population and upon the ability to reposition the probe at exactly the same point on two successive measurements. Failure to reposition accurately will result in a smaller covariance.

Now equation (9) may be

$$\begin{aligned}\mathrm{Var}(m_1 - m_2) \doteq b^2\Bigg(&\mathrm{Var}(L_1) + \mathrm{Var}(L_2) \\ &+ \frac{\mathrm{Var}(I_1)}{T_1} + \frac{\mathrm{Var}(I_2)}{T_2} + \frac{C_1^2\,\mathrm{Var}(t)}{T_1^2} \\ &+ \frac{C_2^2\,\mathrm{Var}(t)}{T_2^2} - 2\,\mathrm{Cov}\,L_1L_2\Bigg)\end{aligned} \tag{10}$$

DISCUSSION

Equations (*8*) and (*10*) can be used to evaluate in theory the increase in precision to be gained by varying the number of sampling locations n and the timing interval T within the limits of the gross amount of time ($\sum T$) available for sampling the area. Some simplifying assumptions will be helpful with reference to actual field data in order to illustrate the relative importance of the three sources of error.

For this purpose the data from 26 random locations at the Union Research Center in the South Carolina Piedmont and 32 random locations at the Coweeta Hydrologic Laboratory in the southern Appalachian mountains in North Carolina are used. The blocks sampled were several acres in size and all samples were obtained in counts per minute with the neutron probe at 2 feet below the surface in forest soil. Moisture content ranged from 25 to 50 per cent by volume, averaging 40 per cent. It was desired to measure the loss or gain in soil moisture over a period of about 14 days and to estimate its variance, $S^2(m_1 - m_2)$, where m_1 is the moisture content on date 1 and m_2 is the moisture content on date 2.

The calibration coefficient converting counts per minute to per cent by volume was $b = 0.004$ for the equipment used (Nuclear-Chicago's Model 2800 scaler and P-19 probe). The variance of the timing interval T, averaged among several observers, was found to be independent of the length of T, and a value Var $(t) = 0.000003$ in minutes is used throughout these comparisons.

Variance due to instrumental error Var (I) in equation (*8*) may be determined from the scaler by making successive counts with the probe in a fixed position in the soil to be sampled, or by calculating the instrument variance according to Jarrett (4). In the symbols selected for use in this paper, Jarrett's results, based on use of the Poisson distribution, showed that Var $(I) = C$, where C is, of course, the population mean of counts per minute for the body of soil sampled. Applying this relationship to equation (*8*), it can be seen that the magnitude of the average moisture content will affect instrument as well as timing variance. Nevertheless, the effect is small unless average moisture content is very high and fluctuates widely between observations and the timing interval T is very short.

TABLE 1

Components of equations (8) and (10)

Total Variance	=	Locational	+ Instrumental	+ Timing
Union Research Center				
m_1	=	52.245	0.160	0.005
m_2	=	49.956	0.160	0.005
$m_1 - m_2$	=	1.984	0.320	0.010
Coweeta Hydrologic Laboratory				
m_1	=	25.052	0.160	0.005
m_2	=	25.862	0.160	0.005
$m_1 - m_2$	=	2.037	0.320	0.010

For the moisture contents investigated here, Var (I) was calculated to be 10,000, that is the standard deviation = 100 counts per minute.

Using this information together with the actual variance observed in the field at the two experimental locations, the components of equations (*8*) and (*10*) assume the values shown in table 1.

Two important conclusions can be drawn from the figures in table 1: the instrument and timing errors are altogether negligible in comparison to location error in a single moisture estimate; and the locational covariance in equation (*10*) greatly reduces the estimate of variance of moisture change between dates 1 and 2. In estimates of moisture change, instrument and timing error become a measurable percentage of the total variance of the difference (14 per cent in this case). Note that slight differences in moisture level between dates have been ignored in calculating Var (I) and C^2 Var (t).

The usual way to reduce error in sampling procedures is to increase the number of observations (n). In using the neutron meter, however, there has been a tendency to overemphasize the importance of extending the timing interval T. Figure 1, representing a solution of equations (*8*) and (*10*), shows by the shaded area that increases in T provide rather unimportant reductions in error, regardless of the size of n. To construct this graph, 5 minutes was arbitrarily chosen as the smallest reasonable total counting time ($\sum T$) available for sampling a soil stratum, and the $\sum T$ was divided by n for use in the equations. Thus, when $n = 2$, $T = 2.5$ minutes, and when $n = 100$, $T = 0.05$ minute.

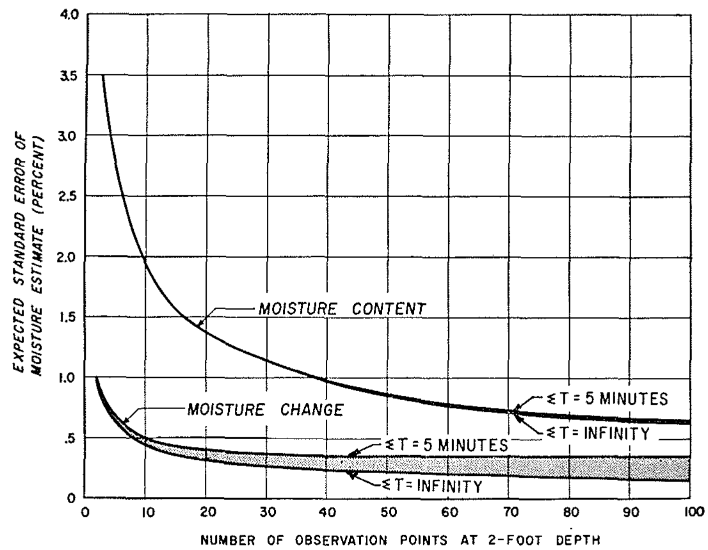

FIG. 1. Relation of the expected standard error of estimate to the number of observations made, on both soil moisture content and moisture change. Shading indicates the loss in precision as the total counting time (ΣT) is reduced from infinity to only 5 minutes (see text). Data averaged from Coweeta and Union.

The equations may be solved and plotted in a variety of ways, but the lower limit of expected error for any n is described, as in Figure 1, by setting $\sum T$ equal to infinity. At this limit, the instrument and timing errors in equations *(8)* and *(10)* reduce to zero, and only the error due to location remains.

It is concluded that instrument and timing errors are practically negligible. In most sampling problems there is little advantage in increasing $\sum T$ or the timing interval T regardless of the number of observation points. The shortest timing interval T consistent with sampling economy will in most cases lie between 10 and 30 seconds. (It should be emphasized again that our problem calls for estimation of mean moisture or moisture change in a stratum. If, on the other hand, plans call for precise information about moisture behavior at a single point in soil, longer counting intervals are both justified and advisable.)

It is interesting to note that the number of installations (n) needed to estimate soil moisture in these strata to a standard error of 1 per cent by volume (0.12 inch per foot of soil depth) is approximately 38, but that 2 or 3 will measure moisture loss or gain to this precision. It appears from these and other studies that, although soils vary greatly from point to point, they tend to lose or gain moisture rather uniformly.

SUMMARY

The variation of soil moisture estimates determined by the neutron-scattering method is examined in theory and related to field data from two research areas. A statistical model is developed to account for error. Instrument and timing errors are shown to contribute insignificantly to the standard error of estimate in sampling studies. Furthermore, their contribution to estimates of moisture change with time is, for all practical purposes, negligible, as long as the timing interval used at each observation exceeds 30 seconds. In most studies even shorter timing intervals may be used safely.

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