or $$M_1 = -P\frac{ab^2}{l^2}$$

and from Eq. 2, $$M_2 = -P\frac{a^2b}{l^2}$$

Example 2. The fixed beam of Fig. 3 is loaded with a triangular load $P = pl/2$. The bending moment area is

$$A_0 = \frac{P}{3}\int_0^l \left(x - \frac{x^3}{l^2}\right) dx = \frac{Pl^2}{12}$$

and its static moment

$$S_0 = \frac{P}{3}\int_0^l \left(x^2 - \frac{x^4}{l^2}\right) dx = \frac{2}{45}Pl^3$$

so that the distances of its center of gravity will be

$$c = \frac{S_0}{A_0} = \frac{8}{15}l, \qquad d = l - \frac{8}{15}l = \frac{7}{15}l$$

Consequently $s_1 = \frac{3}{15}l$, and $s_2 = \frac{2}{15}l$, so that by Eq. 1,

$$M_2 = \frac{3}{2}M_1 \ldots\ldots\ldots\ldots\ldots [3]$$

The equality of the moment areas gives

$$\frac{Pl^2}{12} = -M_1\frac{l}{2} - M_1\frac{3}{2}\frac{l}{2} = -M_1\frac{5}{4}l$$

or $$M_1 = -\frac{Pl}{15}$$

and from Eq. 3, $$M_2 = -\frac{Pl}{10}$$

The method described herein is especially useful in beams with complicated loadings. In such cases, after the moment area of the freely supported beam for the given loading has been found, c and d can be determined graphically or analytically by well-known methods.

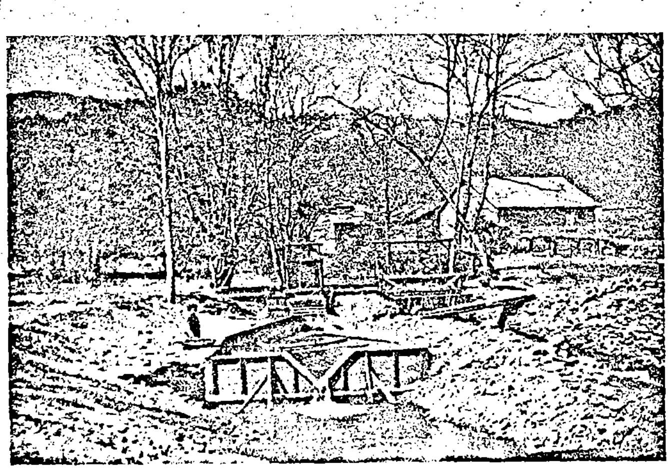
Determination of a Formula for the 120-Deg V-Notch Weir

By R. A. HERTZLER, JUN. AM. SOC. C.E.

SUPERINTENDENT, COWEETA EXPERIMENTAL FOREST, APPALACHIAN FOREST EXPERIMENT STATION, U. S. FOREST SERVICE

ONE phase of the "forest influences" investigations now being made by the Appalachian Forest Experiment Station is a study of the effect of vegetative cover and land use on stream flow. As basic data for this work, the maximum, minimum, and normal discharges, as well as the total yield, of streams must be determined for drainage areas of known vegetative cover. To obtain these data, various types of stream gaging installations are used. As silt is a minor factor in streams from forested areas located on the old granitic formations of the Southern Appalachian Mountains, a sharp-crested weir equipped with a recorder which produces a continuous stage discharge hydrograph is well adapted for use in this region. From this chart ground-water flow and surface runoff are computed and compiled for further analysis.

The 90-deg V-notch weir is limited to low flows, with high accuracy for the minimum discharge. On the other hand, rectangular and Cipoletti weirs are better adapted to the higher flows, but are not suited to the minimum flows of the streams in these areas. It was therefore necessary for this work that a weir be designed to measure stream flow from zero to 26 cu ft per sec, the minimum and maximum flows estimated from areas ranging in size from 5 to 150 acres, and the 120-deg V-notch weir was found to meet the requirements.



COWEETA EXPERIMENTAL FOREST TESTING STATION
A 90-Deg V-Notch Weir Is Shown in Tandem with a Suppressed Rectangular Weir

A comparison of the formula for the 120-deg V-notch weir, as determined here, with formulas for the 90-deg V-notch and rectangular weirs shows that for a 2-ft head the capacity of the 120-deg blade is 1.73 times that of the 90-deg blade and slightly greater than the capacity of a 2.6-ft rectangular blade. The adopted design is shown in Fig. 1.

Previous tests conducted with 120-deg V-notch weirs by Cone, Gourley and Crimp, Tarrant, Greve, and Doebler and Rayfield, ("Flow of Water Through Circular, Parabolic and Triangular Vertical Notch Weirs," by F. W. Greve, Engineering Bulletin of Purdue University, Research Series No. 40, March 1932) had resulted in the following discharge formulas:

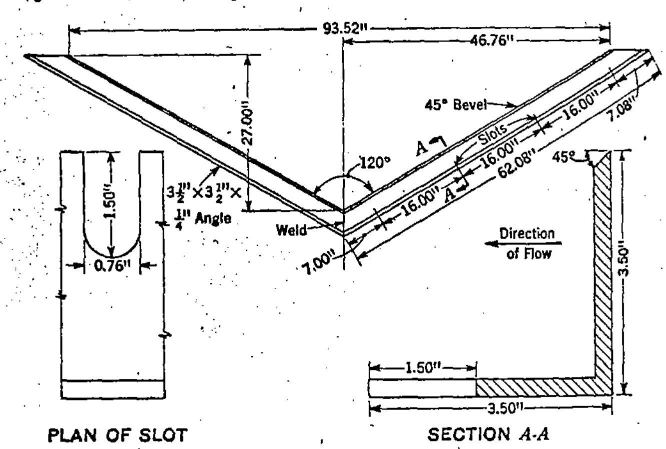


FIG. 1. 120-DEG V-NOTCH WEIR BLADE

Cone	$Q = 4.40H^{2.487}$
Tarrant	$Q = 4.312H^{2.485}$
Gourley and Crimp	$Q = 4.295H^{2.470}$
Greve	$Q = 4.326H^{2.470}$
Doebler and Rayfield	$Q = 4.38H^{2.500}$

However, these formulas were not derived for the conditions under which several of our installations were operating. Accordingly, calibration tests of a specially designed blade were made in the field hydraulics laboratory of the Coweeta Experimental Forest, near Franklin, N.C., by comparison with discharges from a 3.95-ft suppressed rectangular weir of known discharge formula.

Both weirs were made of structural steel angles, fabricated to within $^1/_{32}$ in., beveled at 45 deg to a sharp edge at the upstream face, painted, and bolted to wooden frames in the control channel. The channel was constructed of surfaced lumber which afforded a minimum frictional resistance.

The stilling basin for the triangular weir was large enough to obtain a minimum side contraction of 2.5 times the head, with a bottom contraction of 2.5 ft. The influent to the stilling basin or weir box, in tandem with the rectangular weir, was baffled with 1-in. by 6-in. adjustable boards in addition to a "wave reducer" consisting of a festoon of floating logs. Air was supplied to the nappe of the suppressed rectangular weir through holes drilled in the side of the channel. Heads over both weirs were measured in wooden stilling wells equipped with hook gages reading to 0.001 ft.

A total of 103 test runs were conducted with free-flow conditions over both weirs. Discharges greater than 0.5 cu ft per sec were determined from the observed head over the rectangular weir. After the flow was regulated through the field laboratory channel by a control gate, steady flow conditions were indicated by five consecutive similar hook gage readings, taken at 5-minute intervals at both weirs. Discharge measurements less than 0.5 cu ft per sec were taken by weight.

As a result of these tests, the following formula was derived for the 120-deg V-notch weir, for discharges greater than 0.2 cu ft per sec:

$$Q = 4.43H^{2.449}$$

where Q is the discharge in cubic feet per second, and H is the observed head measured 6 ft upstream from the blade. This formula indicates a slightly higher discharge than do those of previous experimenters, whose work was done in channels of limited depth or width.

Our Readers Say—

In Comment on Papers, Society Affairs, and Related Professional Interests

Formulas for Deflections of Cantilevers

DEAR SIR: Equations 1, 2, and 3, presented by R. Reuben Kohn in his article in the July issue, give reliable results when the cantilever is divided into equal lengths, and when that condition exists the formulas would have to be classed as special formulas. For a concentrated load at the end of a cantilever and with the moment of inertia changing along its length, it seems to me that the algebraic moment-area method is as general as any, and the results can be obtained in a very short time. Using the values given by Mr. Kohn with 360 lb applied at the end, I get a deflection of 36.8 in. by the algebraic moment-area method.

For a cantilever with a uniform load applied along its length and having the moment of inertia changing along its length the deflection can be solved by the slope-deflection method, which can be classed as a general method.

Using the author's example with a wind load of 27 lb per sq ft on the projected area of the cantilever, the moments can be calculated due to cantilever action at the various joints where the moment of inertia changes in value. In the accompanying Fig. 1 these moments are indicated in parentheses.

Now assume a hinged support (which is not there) at each joint where the moment of inertia changes in value and then calculate the fixed-end moments due to the uniform loads at the various joints. These moments are indicated in Fig. 1 in brackets. At the end where the cantilever is fixed the angle of rotation, θ_1, is zero, and applying the slope-deflection equation for the first section

$$-1{,}030{,}000 = 2E(2.716)\ (0 + \theta_2 - 3R_1) - 9{,}464 \quad [1]$$

$$\text{From Eq. 1, } E\theta_2 = -187{,}900 + 3ER_1 \quad [2]$$

$$\text{Also, } +649{,}000 = 2E(2.716)\ (-375{,}800 + 6R_1 + 0 - 3R_1) + 9{,}464 \quad [3]$$

From Eq. 3, $3ER_1 = 493{,}600$; thus from Eq. 2, $E\theta_2 = 305{,}700$.

Repeating this operation until the end of the cantilever is reached, all the θ's and R's can be solved. Their values are shown in Fig. 1.

Adding all the R's we get $\frac{5{,}020{,}900}{E}$, and as each length represents 208 in. the deflection at the end of the cantilever will be $\frac{5{,}020{,}900 \times 208}{29{,}000{,}000} = 36.01$ in. The author with his assumed condition gets 38.8 in., which he states to be a trifle large.

By means of the slope-deflection method the displacement at any joint can readily be solved. From Fig. 1 the displacement of the joint between the 14- and 12-in. pipe is $R_1 \times 208 = \frac{164{,}500 \times 208}{29{,}000{,}000} = 1.18$ in., and the displacement of the joint between the 12- and 10-in. pipe is $208(R_1 + R_2) = 3.37$ in.

When the cantilever under consideration is divided into equal lengths, Mr. Kohn's method will give results that are a trifle large, and the displacement can be solved in a very short time. If the lengths are not equal then his three special equations cannot be used. On the other hand, by using the slope-deflection method

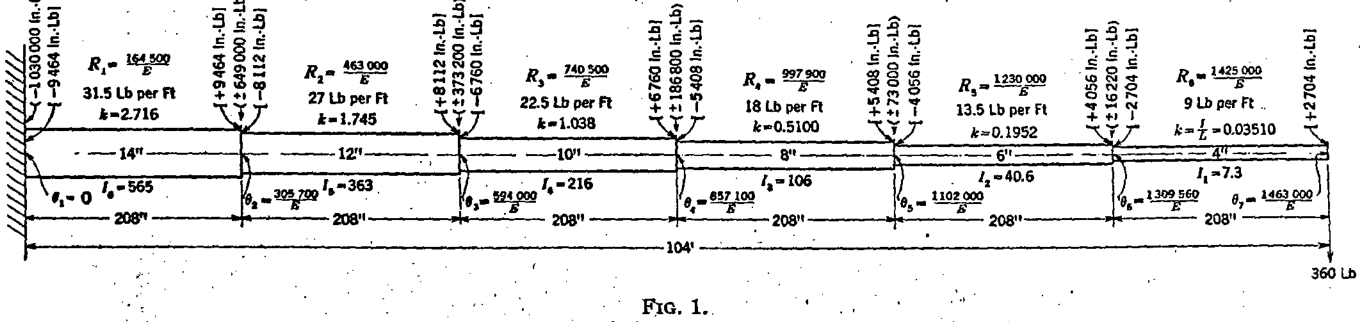


FIG. 1.